A Review of Fractions

Learning Objectives

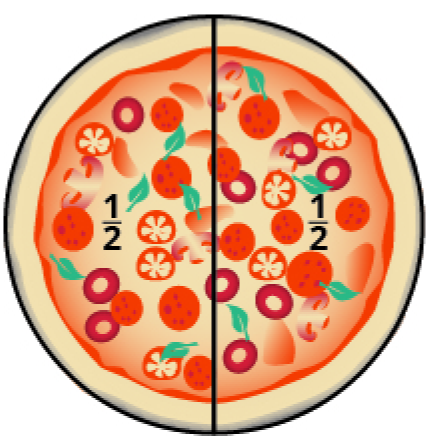
By the end of this section, you will be able to:

- Understand the meaning of fractions
- Model improper fractions and mixed numbers
- Convert between improper fractions and mixed numbers
- Model equivalent fractions
- Find equivalent fractions
- Locate fractions and mixed numbers on the number line
- Order fractions and mixed numbers

Understand the Meaning of Fractions

Andy and Bobby love pizza. On Monday night, they share a pizza equally. How much of the pizza does each one get? Are you thinking that each boy gets half of the pizza? That's right. There is one whole pizza, evenly divided into two parts, so each boy gets one of the two equal parts.

In math, we write $\frac{1}{2}$ to mean one out of two parts.



On Tuesday, Andy and Bobby share a pizza with their parents, Fred and

Christy, with each person getting an equal amount of the whole pizza. How much of the pizza does each person get? There is one whole pizza, divided evenly into four equal parts. Each person has one of the four equal parts, so each has $\frac{1}{4}$ of the pizza.

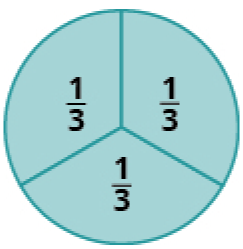
On Wednesday, the family invites some friends over for a pizza dinner. There are a total of 12 people. If they share the pizza equally, each person would get $\frac{1}{12}$ of the pizza.

Note:
Fractions

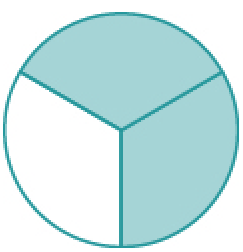
A fraction is written $\frac{a}{b}$, where a and b are integers and $b \neq 0$. In a fraction, a is called the numerator and b is called the denominator.

A fraction is a way to represent parts of a whole. The denominator b represents the number of equal parts the whole has been divided into, and the numerator a represents how many parts are included. The denominator, b, cannot equal zero because division by zero is undefined.

In [link], the circle has been divided into three parts of equal size. Each part represents $\frac{1}{3}$ of the circle. This type of model is called a fraction circle. Other shapes, such as rectangles, can also be used to model fractions.



What does the fraction $\frac{2}{3}$ represent? The fraction $\frac{2}{3}$ means two of three equal parts.

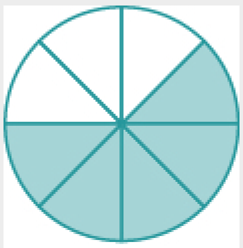


Example:

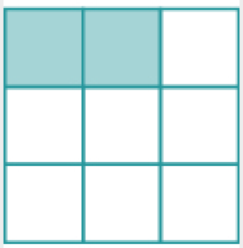
Exercise:

Problem:

Name the fraction of the shape that is shaded in each of the figures.



(a)



(b)

Solution:

We need to ask two questions. First, how many equal parts are there? This will be the denominator. Second, of these equal parts, how many are shaded? This will be the numerator.

(a)

How many equal parts are there?	There are eight equal parts.
How many are shaded?	Five parts are shaded.

Five out of eight parts are shaded. Therefore, the fraction of the circle that is shaded is $\frac{5}{8}$.

(b)

How many equal parts are there? There are nine equal parts.

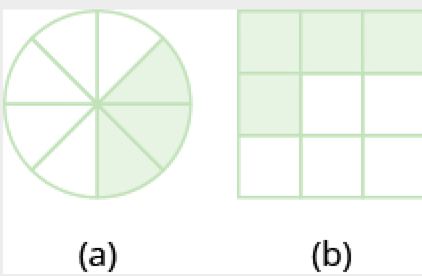
How many are shaded? Two parts are shaded.

Two out of nine parts are shaded. Therefore, the fraction of the square that is shaded is $\frac{2}{9}$.

Note:
Try It
Exercise:

Problem:

Name the fraction of the shape that is shaded in each figure:



Solution:

$\frac{3}{8}$
$\frac{4}{9}$

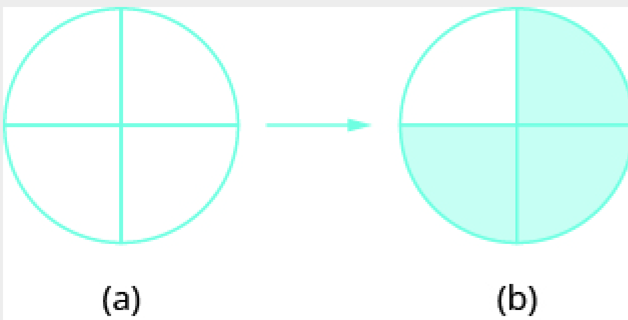
Example:
Exercise:

Problem: Shade $\frac{3}{4}$ of the circle.

Solution:

The denominator is 4, so we divide the circle into four equal parts ⓐ.

The numerator is 3, so we shade three of the four parts ⓑ.

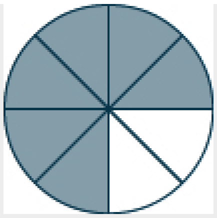


$\frac{3}{4}$ of the circle is shaded.

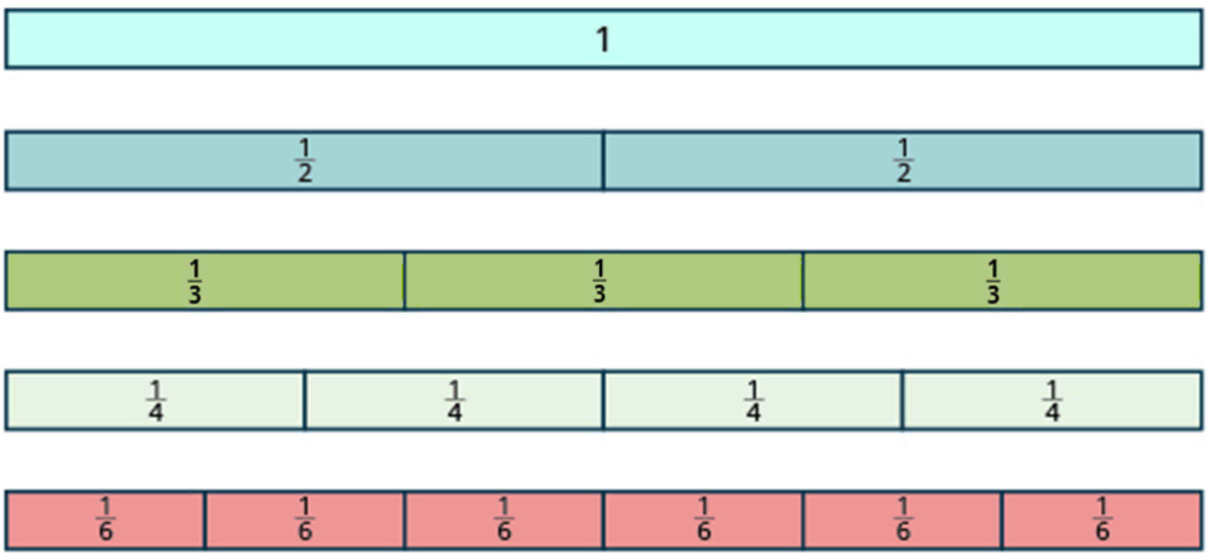
Note:
Try It
Exercise:

Problem: Shade $\frac{6}{8}$ of a circle.

Solution:



In [link] and [link], we used circles and rectangles to model fractions. Fractions can also be modeled as manipulatives called fraction tiles, as shown in [link]. Here, the whole is modeled as one long, undivided rectangular tile. Beneath it are tiles of equal length divided into different numbers of equally sized parts.



We'll be using fraction tiles to discover some basic facts about fractions. Refer to [link] to answer the following questions:

How many $\frac{1}{2}$ tiles does it take to make one whole tile?

It takes two halves to make a whole, so two out of two is $\frac{2}{2} = 1$.

How many $\frac{1}{3}$ tiles does it take to make one whole tile?

It takes three thirds, so three out of three is $\frac{3}{3} = 1$.

How many $\frac{1}{4}$ tiles does it take to make one whole tile?

It takes four fourths, so four out of four is $\frac{4}{4} = 1$.

How many $\frac{1}{6}$ tiles does it take to make one whole tile?

It takes six sixths, so six out of six is $\frac{6}{6} = 1$.

What if the whole were divided into 24 equal parts? (We have not shown fraction tiles to represent this, but try to visualize it in your mind.) How many $\frac{1}{24}$ tiles does it take to make one whole tile?

It takes 24 twenty-fourths, so $\frac{24}{24} = 1$.

This leads us to the *Property of One*.

Note:
Property of One
Any number, except zero, divided by itself is one.
Equation:

$$\frac{a}{a} = 1 \qquad (a \neq 0)$$

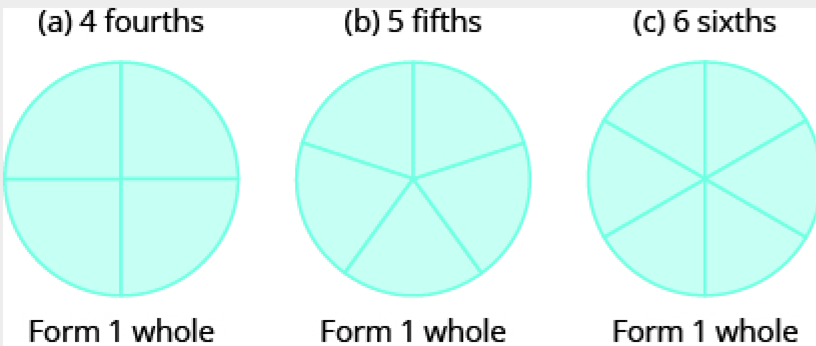
Example:
Exercise:

Problem:

Use fraction circles to make wholes using the following pieces:

4 fourths
5 fifths
6 sixths

Solution:

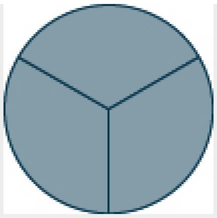


Note:
Try It
Exercise:

Problem:

Use fraction circles to make wholes with the following pieces: 3 thirds.

Solution:



What if we have more fraction pieces than we need for 1 whole? We'll look at this in the next example.

Example:
Exercise:

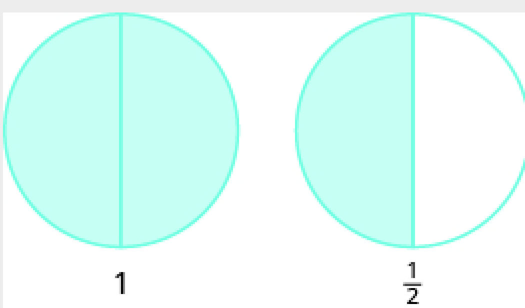
Problem:

Use fraction circles to make wholes using the following pieces:

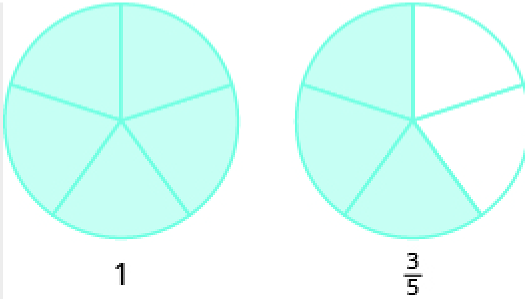
3 halves
8 fifths
7 thirds

Solution:

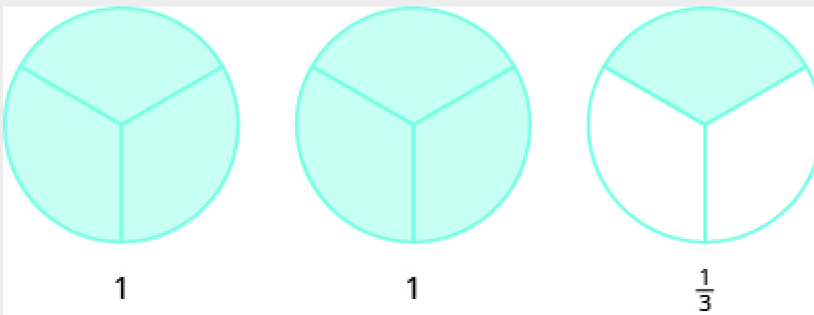
a. 3 halves make 1 whole with 1 half left over.



b. 8 fifths make 1 whole with 3 fifths left over.



c. 7 thirds make 2 wholes with 1 third left over.

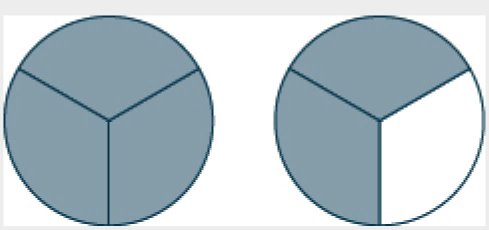


Note:
Try It
Exercise:

Problem:

Use fraction circles to make wholes with the following pieces: 5 thirds.

Solution:



Model Improper Fractions and Mixed Numbers

In [link] (b), you had eight equal fifth pieces. You used five of them to make one whole, and you had three fifths left over. Let us use fraction notation to show what happened. You had eight pieces, each of them one fifth, $\frac{1}{5}$, so altogether you had eight fifths, which we can write as $\frac{8}{5}$. The fraction $\frac{8}{5}$ is one whole, 1, plus three fifths, $\frac{3}{5}$, or $1\frac{3}{5}$, which is read as *one and three-fifths*.

The number $1\frac{3}{5}$ is called a mixed number. A mixed number consists of a whole number and a fraction.

Note:
Mixed Numbers
A **mixed number** consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$. It is written as follows.
Equation:

$$a\frac{b}{c} \qquad c \neq 0$$

Fractions such as $\frac{5}{4}$, $\frac{3}{2}$, $\frac{5}{5}$, and $\frac{7}{3}$ are called improper fractions. In an improper fraction, the numerator is greater than or equal to the denominator, so its value is greater than or equal to one. When a fraction has a numerator that is smaller than the denominator, it is called a proper fraction, and its value is less than one. Fractions such as $\frac{1}{2}$, $\frac{3}{7}$, and $\frac{11}{18}$ are proper fractions.

Note:
Proper and Improper Fractions
The fraction $\frac{a}{b}$ is a **proper fraction** if $a < b$ and an **improper fraction** if $a \geq b$.

Example:
Exercise:

Problem:

Name the improper fraction modeled. Then write the improper fraction as a mixed number.

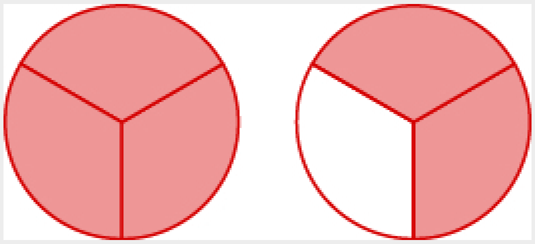
Solution:

Each circle is divided into three pieces, so each piece is $\frac{1}{3}$ of the circle. There are four pieces shaded, so there are four thirds or $\frac{4}{3}$. The figure shows that we also have one whole circle and one third, which is $1\frac{1}{3}$. So, $\frac{4}{3} = 1\frac{1}{3}$.

Note:
Try It
Exercise:

Problem:

Name the improper fraction. Then write it as a mixed number.



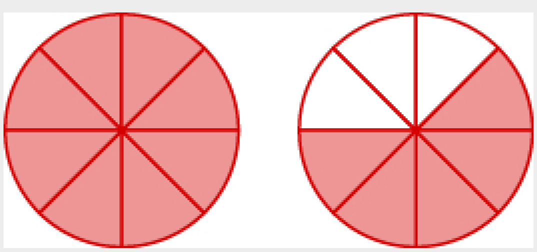
Solution:

$\frac{5}{3} = 1\frac{2}{3}$

Note:
Try It
Exercise:

Problem:

Name the improper fraction. Then write it as a mixed number.



Solution:

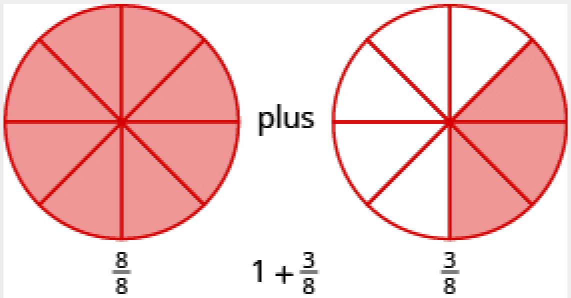
$\frac{13}{8} = 1\frac{5}{8}$

Example:
Exercise:

Problem: Draw a figure to model $\frac{11}{8}$.

Solution:

The denominator of the improper fraction is 8. Draw a circle divided into eight pieces and shade all of them. This takes care of eight eighths, but we have 11 eighths. We must shade three of the eight parts of another circle.

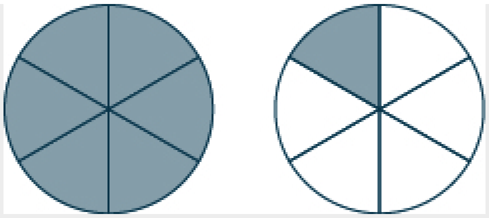


So, $\frac{11}{8} = 1\frac{3}{8}$.

Note:
Try It
Exercise:

Problem: Draw a figure to model $\frac{7}{6}$.

Solution:



Example:
Exercise:

Problem:

Use a model to rewrite the improper fraction $\frac{11}{6}$ as a mixed number.

Solution:

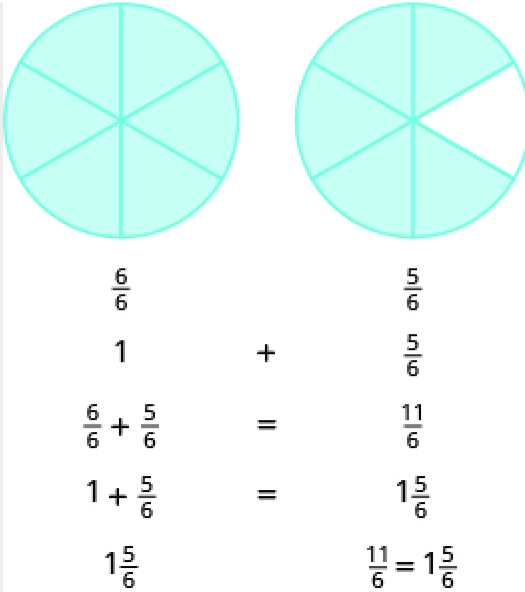
We start with 11 sixths $\left(\frac{11}{6}\right)$. We know that six sixths makes one whole.

Equation:

$$\frac{6}{6} = 1$$

That leaves us with five more sixths, which is $\frac{5}{6}$ $\left(11 \text{ sixths minus } 6 \text{ sixths is } 5 \text{ sixths}\right)$.

So, $\frac{11}{6} = 1\frac{5}{6}$.



Note:
Try It
Exercise:

Problem:

Use a model to rewrite the improper fraction as a mixed number: $\frac{9}{7}$.

Solution:

$1\frac{2}{7}$

Example:
Exercise:

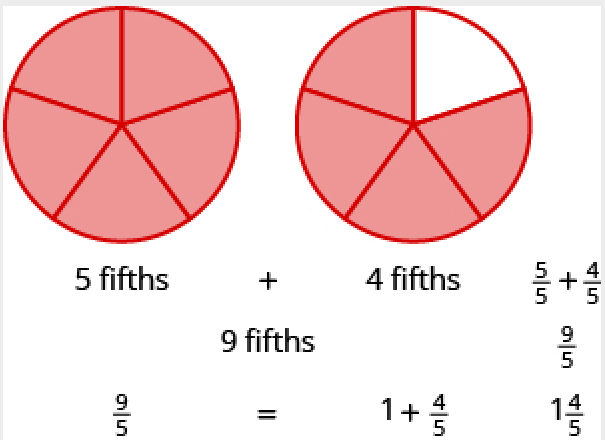
Problem:

Use a model to rewrite the mixed number $1\frac{4}{5}$ as an improper fraction.

Solution:

The mixed number $1\frac{4}{5}$ means one whole plus four fifths. The denominator is 5, so the whole is $\frac{5}{5}$. Together five fifths and four fifths equals nine fifths.

So, $1\frac{4}{5} = \frac{9}{5}$.



Note:
Try It
Exercise:

Problem:

Use a model to rewrite the mixed number as an improper fraction: $1\frac{3}{8}$.

Solution:

$\frac{11}{8}$

Convert between Improper Fractions and Mixed Numbers

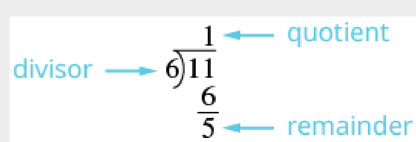
In [link], we converted the improper fraction $\frac{11}{6}$ to the mixed number $1\frac{5}{6}$ using fraction circles. We did this by grouping six sixths together to make a whole; then we looked to see how many of the 11 pieces were left. We saw that $\frac{11}{6}$ made one whole group of six sixths plus five more sixths, showing that $\frac{11}{6} = 1\frac{5}{6}$.

The division expression $\frac{11}{6}$ (which can also be written as $6\overline{)11}$) tells us to find how many groups of 6 are in 11. To convert an improper fraction to a mixed number without fraction circles, we divide.

Example:
Exercise:

Problem: Convert $\frac{11}{6}$ to a mixed number.

Solution:

	$\frac{11}{6}$
Divide the denominator into the numerator.	Remember $\frac{11}{6}$ means $11 \div 6$.
	

Identify the quotient, remainder and divisor.	
Write the mixed number as $\frac{\text{remainder}}{\text{divisor}}$.	$1\frac{5}{6}$
So, $\frac{11}{6} = 1\frac{5}{6}$	

Note:
Try It
Exercise:

Problem: Convert the improper fraction to a mixed number: $\frac{13}{7}$.

Solution:

$1\frac{6}{7}$.

Note:
Try It
Exercise:

Problem: Convert the improper fraction to a mixed number: $\frac{14}{9}$.

Solution:

$1\frac{5}{9}$

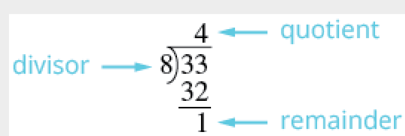
Note:
Convert an improper fraction to a mixed number.

Divide the denominator into the numerator.
Identify the quotient, remainder, and divisor.
Write the mixed number as quotient $\frac{\text{remainder}}{\text{divisor}}$.

Example:
Exercise:

Problem: Convert the improper fraction $\frac{33}{8}$ to a mixed number.

Solution:

	$\frac{33}{8}$
Divide the denominator into the numerator.	Remember, $\frac{33}{8}$ means $8\overline{)33}$.
Identify the quotient, remainder, and divisor.	
Write the mixed number as quotient $\frac{\text{remainder}}{\text{divisor}}$.	$4\frac{1}{8}$

	So, $\frac{33}{8} = 4\frac{1}{8}$

Note:
Try It
Exercise:

Problem: Convert the improper fraction to a mixed number: $\frac{23}{7}$.

Solution:

$3\frac{2}{7}$

Note:
Try It
Exercise:

Problem: Convert the improper fraction to a mixed number: $\frac{48}{11}$.

Solution:

$4\frac{4}{11}$

In [link], we changed $1\frac{4}{5}$ to an improper fraction by first seeing that the whole is a set of five fifths. So we had five fifths and four more fifths.
Equation:

$$\frac{5}{5} + \frac{4}{5} = \frac{9}{5}$$

Where did the nine come from? There are nine fifths—one whole (five fifths) plus four fifths. Let us use this idea to see how to convert a mixed number to an improper fraction.

Example:
Exercise:

Problem: Convert the mixed number $4\frac{2}{3}$ to an improper fraction.

Solution:

	$4\frac{2}{3}$
Multiply the whole number by the denominator.	
The whole number is 4 and the denominator is 3.	$\frac{4 \cdot 3 + \square}{\square}$
Simplify.	$\frac{12 + \square}{\square}$
Add the numerator to the product.	

The numerator of the mixed number is 2.	$\frac{12 + 2}{\square}$
Simplify.	$\frac{14}{\square}$
Write the final sum over the original denominator.	
The denominator is 3.	$\frac{14}{3}$

Note:
Try It
Exercise:

Problem: Convert the mixed number to an improper fraction: $3\frac{5}{7}$.

Solution:

$\frac{26}{7}$

Note:
Try It
Exercise:

Problem: Convert the mixed number to an improper fraction: $2\frac{7}{8}$.

Solution:

$\frac{23}{8}$

Note:
Convert a mixed number to an improper fraction

Multiply the whole number by the denominator.
Add the numerator to the product found in Step 1.
Write the final sum over the original denominator.

Example:
Exercise:

Problem: Convert the mixed number $10\frac{2}{7}$ to an improper fraction.

Solution:

	$10\frac{2}{7}$
Multiply the whole number by the denominator.	
The whole number is 10 and the denominator is 7.	$\frac{10 \cdot 7 + \square}{\square}$

Simplify.	$\frac{70 + \square}{\square}$
Add the numerator to the product.	
The numerator of the mixed number is 2.	$\frac{70 + 2}{\square}$
Simplify.	$\frac{72}{\square}$
Write the final sum over the original denominator.	
The denominator is 7.	$\frac{72}{7}$

Note:
Try It
Exercise:

Problem: Convert the mixed number to an improper fraction: $4\frac{6}{11}$.

Solution:

$\frac{50}{11}$

Note:
Try It
Exercise:

Problem: Convert the mixed number to an improper fraction: $11\frac{1}{3}$.

Solution:

$\frac{34}{3}$

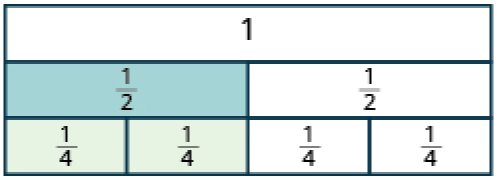
Model Equivalent Fractions

Let's think about Andy and Bobby and their favorite food again. If Andy eats $\frac{1}{2}$ of a pizza and Bobby eats $\frac{2}{4}$ of the pizza, have they eaten the same amount of pizza? In other words, does $\frac{1}{2} = \frac{2}{4}$? We can use fraction tiles to find out whether Andy and Bobby have eaten *equivalent* parts of the pizza.

Note:
Equivalent Fractions
Equivalent fractions are fractions that have the same value.

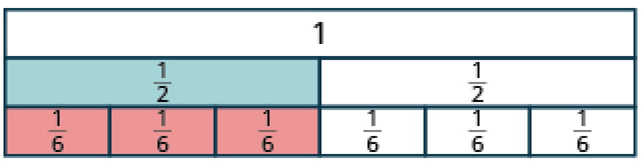
Fraction tiles serve as a useful model of equivalent fractions. You may want to use fraction tiles to do the following activity. Or you might make a copy of [link] and extend it to include eighths, tenths, and twelfths.

Start with a $\frac{1}{2}$ tile. How many fourths equal one-half? How many of the $\frac{1}{4}$ tiles exactly cover the $\frac{1}{2}$ tile?



Since two $\frac{1}{4}$ tiles cover the $\frac{1}{2}$ tile, we see that $\frac{2}{4}$ is the same as $\frac{1}{2}$, or $\frac{2}{4} = \frac{1}{2}$.

How many of the $\frac{1}{6}$ tiles cover the $\frac{1}{2}$ tile?



Since three $\frac{1}{6}$ tiles cover the $\frac{1}{2}$ tile, we see that $\frac{3}{6}$ is the same as $\frac{1}{2}$.

So, $\frac{3}{6} = \frac{1}{2}$. The fractions are equivalent fractions.

Example:
Exercise:

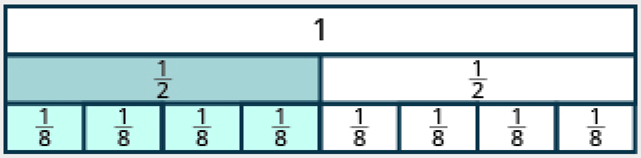
Problem:

Use fraction tiles to find equivalent fractions. Show your result with a figure.

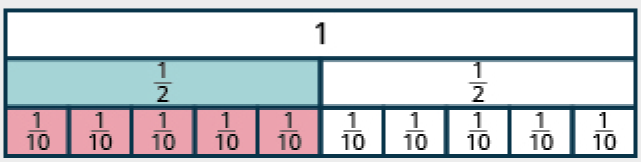
How many eighths equal one-half?
How many tenths equal one-half?
How many twelfths equal one-half?

Solution:

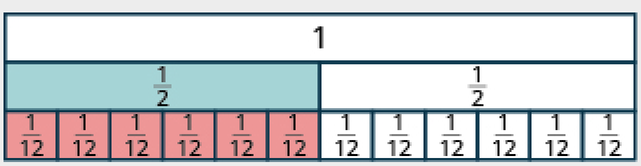
a. It takes four $\frac{1}{8}$ tiles to exactly cover the $\frac{1}{2}$ tile, so $\frac{4}{8} = \frac{1}{2}$.



b. It takes five $\frac{1}{10}$ tiles to exactly cover the $\frac{1}{2}$ tile, so $\frac{5}{10} = \frac{1}{2}$.



c. It takes six $\frac{1}{12}$ tiles to exactly cover the $\frac{1}{2}$ tile, so $\frac{6}{12} = \frac{1}{2}$.



Suppose you had tiles marked $\frac{1}{20}$. How many of them would it take to equal $\frac{1}{2}$? Are you thinking ten tiles? If you are, you’re right, because $\frac{10}{20} = \frac{1}{2}$.

We have shown that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$, and $\frac{10}{20}$ are all equivalent fractions.

Note:
Try It
Exercise:

Problem:

Use fraction tiles to find equivalent fractions: How many eighths equal one-fourth?

Solution:

2

Note:
Try It
Exercise:

Problem:

Use fraction tiles to find equivalent fractions: How many twelfths equal one-fourth?

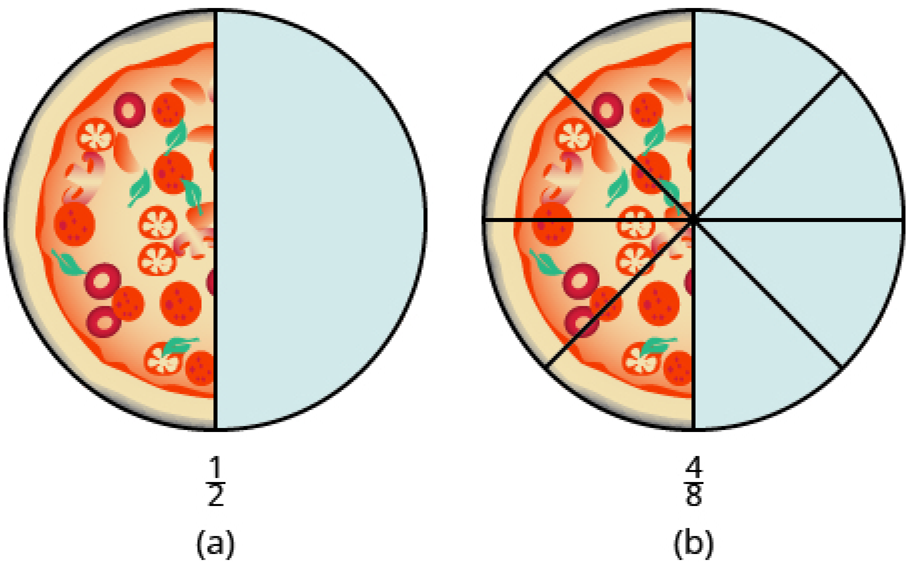
Solution:

3

Find Equivalent Fractions

We used fraction tiles to show that there are many fractions equivalent to $\frac{1}{2}$. For example, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ are all equivalent to $\frac{1}{2}$. When we lined up the fraction tiles, it took four of the $\frac{1}{8}$ tiles to make the same length as a $\frac{1}{2}$ tile. This showed that $\frac{4}{8}=\frac{1}{2}$. See [link].

We can show this with pizzas, too. [link](a) shows a single pizza, cut into two equal pieces with $\frac{1}{2}$ shaded. [link](b) shows a second pizza of the same size, cut into eight pieces with $\frac{4}{8}$ shaded.



This is another way to show that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$.

How can we use mathematics to change $\frac{1}{2}$ into $\frac{4}{8}$? How could you take a pizza that is cut into two pieces and cut it into eight pieces? You could cut each of the two larger pieces into four smaller pieces! The whole pizza would then be cut into eight pieces instead of just two. Mathematically, what we've described could be written as:

$$\frac{1 \cdot 4}{2 \cdot 4}=\frac{4}{8}$$

These models lead to the Equivalent Fractions Property, which states that if we multiply the numerator and denominator of a fraction by the same number, the value of the fraction does not change.

Note:
Equivalent Fractions Property
If a, b, and c are numbers where $b \neq 0$ and $c \neq 0$, then
Equation:

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$$

When working with fractions, it is often necessary to express the same fraction in different forms. To find equivalent forms of a fraction, we can use the Equivalent Fractions Property. For example, consider the fraction one-half.

$$\frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6} \quad \text{so} \quad \frac{1}{2} = \frac{3}{6}$$

$$\frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4} \quad \text{so} \quad \frac{1}{2} = \frac{2}{4}$$

$$\frac{1 \cdot 10}{2 \cdot 10} = \frac{10}{20} \quad \text{so} \quad \frac{1}{2} = \frac{10}{20}$$

So, we say that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{10}{20}$ are equivalent fractions.

Example:
Exercise:

Problem: Find three fractions equivalent to $\frac{2}{5}$.

Solution:

To find a fraction equivalent to $\frac{2}{5}$, we multiply the numerator and denominator by the same number (but not zero). Let us multiply them by 2, 3, and 5.

$$\frac{2\cdot 2}{5\cdot 2}=\frac{4}{10} \qquad \frac{2\cdot 3}{5\cdot 3}=\frac{6}{15} \qquad \frac{2\cdot 5}{5\cdot 5}=\frac{10}{25}$$

So, $\frac{4}{10}$, $\frac{6}{15}$, and $\frac{10}{25}$ are equivalent to $\frac{2}{5}$.

Note:
Try It
Exercise:

Problem: Find three fractions equivalent to $\frac{3}{5}$.

Solution:

Correct answers include $\frac{6}{10}$, $\frac{9}{15}$, and $\frac{12}{20}$.

Note:
Try It

Exercise:

Problem: Find three fractions equivalent to $\frac{4}{5}$.

Solution:

Correct answers include $\frac{8}{10}$, $\frac{12}{15}$, and $\frac{16}{20}$.

Example:

Exercise:

Problem:

Find a fraction with a denominator of 21 that is equivalent to $\frac{2}{7}$.

Solution:

To find equivalent fractions, we multiply the numerator and denominator by the same number. In this case, we need to multiply the denominator by a number that will result in 21.

Since we can multiply 7 by 3 to get 21, we can find the equivalent fraction by multiplying both the numerator and denominator by 3.

$$\frac{2}{7} = \frac{2 \cdot 3}{7 \cdot 3} = \frac{6}{21}$$

Note:

Try It

Exercise:

Problem:

Find a fraction with a denominator of 21 that is equivalent to $\frac{6}{7}$.

Solution:

$\frac{18}{21}$

Note:
Try It
Exercise:

Problem:

Find a fraction with a denominator of 100 that is equivalent to $\frac{3}{10}$.

Solution:

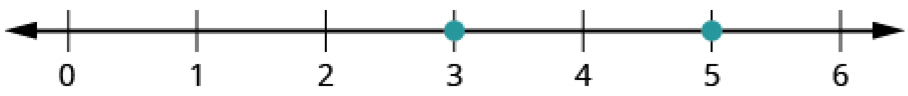
$\frac{30}{100}$

Locate Fractions and Mixed Numbers on the Number Line

Now we are ready to plot fractions on a number line. This will help us visualize fractions and understand their values.

Let us locate $\frac{1}{5}$, $\frac{4}{5}$, 3, $3\frac{1}{3}$, $\frac{7}{4}$, $\frac{9}{2}$, 5, and $\frac{8}{3}$ on the number line.

We will start with the whole numbers 3 and 5 because they are the easiest to plot.



The proper fractions listed are $\frac{1}{5}$ and $\frac{4}{5}$. We know proper fractions have values less than one, so $\frac{1}{5}$ and $\frac{4}{5}$ are located between the whole numbers 0 and 1. The denominators are both 5, so we need to divide the segment of the number line between 0 and 1 into five equal parts. We can do this by drawing four equally spaced marks on the number line, which we can then label as $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$.

Now plot points at $\frac{1}{5}$ and $\frac{4}{5}$.

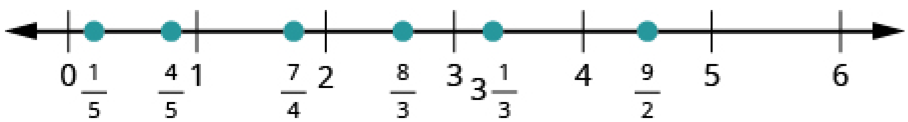
The only mixed number to plot is $3\frac{1}{3}$. Between what two whole numbers is $3\frac{1}{3}$? Remember that a mixed number is a whole number plus a proper fraction, so $3\frac{1}{3} > 3$. Since it is greater than 3, but not a whole unit greater, $3\frac{1}{3}$ is between 3 and 4. We need to divide the portion of the number line between 3 and 4 into three equal pieces (thirds) and plot $3\frac{1}{3}$ at the first mark.

Finally, look at the improper fractions $\frac{7}{4}$, $\frac{9}{2}$, and $\frac{8}{3}$. Locating these points will be easier if you change each of them to a mixed number.

Equation:

$$\frac{7}{4} = 1\frac{3}{4}, \quad \frac{9}{2} = 4\frac{1}{2}, \quad \frac{8}{3} = 2\frac{2}{3}$$

Here is the number line with all the points plotted.



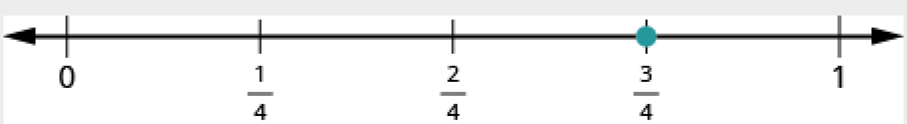
Example:
Exercise:

Problem:

Locate and label the following on a number line: $\frac{3}{4}$, $\frac{4}{3}$, $\frac{5}{3}$, $4\frac{1}{5}$, and $\frac{7}{2}$.

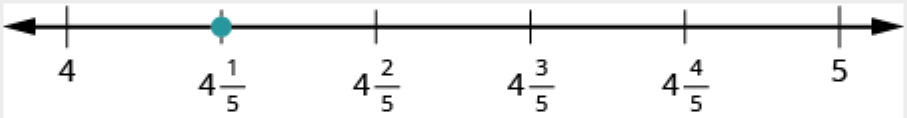
Solution:

Start by locating the proper fraction $\frac{3}{4}$. It is between 0 and 1. To do this, divide the distance between 0 and 1 into four equal parts. Then plot $\frac{3}{4}$.



Next, locate the mixed number $4\frac{1}{5}$. It is between 4 and 5 on the number line. Divide the number line between 4 and 5 into five equal

parts, and then plot $4\frac{1}{5}$ one-fifth of the way between 4 and 5.



Now locate the improper fractions $\frac{4}{3}$ and $\frac{5}{3}$.

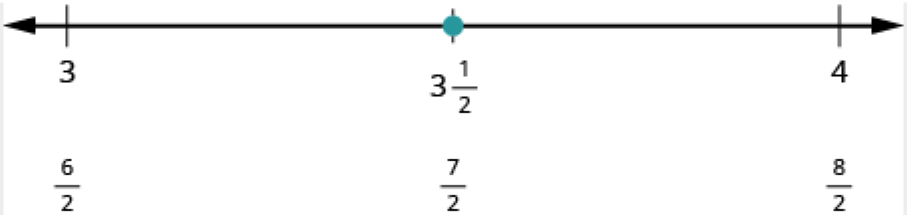
It is easier to plot them if we convert them to mixed numbers first.

Equation:

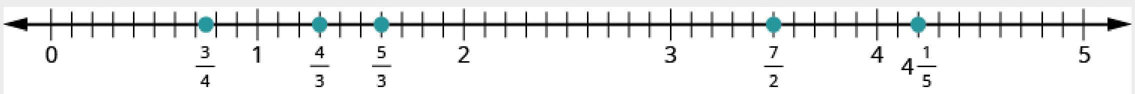
$$\frac{4}{3} = 1\frac{1}{3}, \quad \frac{5}{3} = 1\frac{2}{3}$$

Divide the distance between 1 and 2 into thirds.

Next let us plot $\frac{7}{2}$. We write it as a mixed number, $\frac{7}{2} = 3\frac{1}{2}$. Plot it between 3 and 4.



The number line shows all the numbers located on the number line.

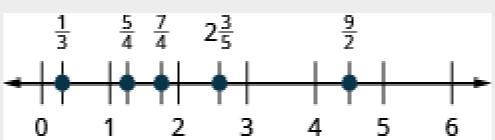


Note:
Try It
Exercise:

Problem:

Locate and label the following on a number line: $\frac{1}{3}, \frac{5}{4}, \frac{7}{4}, 2\frac{3}{5}, \frac{9}{2}$.

Solution:



Order Fractions and Mixed Numbers

We can use the inequality symbols to order fractions. Remember that $a > b$ means that a is to the right of b on the number line. As we move from left to right on a number line, the values increase.

Example:
Exercise:

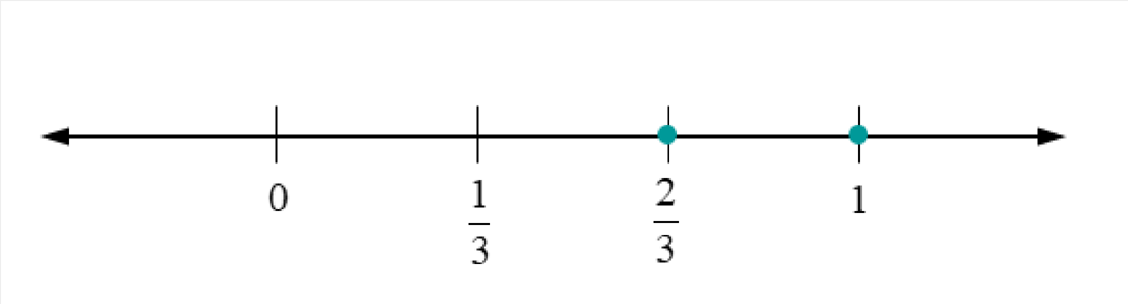
Problem:

Order each of the following pairs of numbers, using $<$ or $>$:

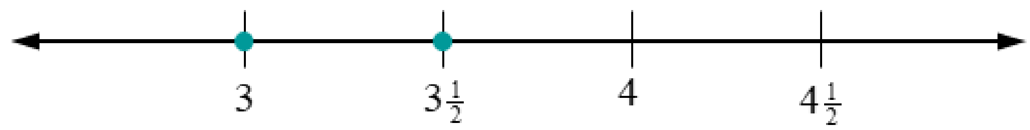
$\frac{2}{3}$ ______ 1
$3\frac{1}{2}$ ______ 3
$\frac{3}{7}$ ______ $\frac{3}{8}$
2 ______ $\frac{16}{9}$

Solution:

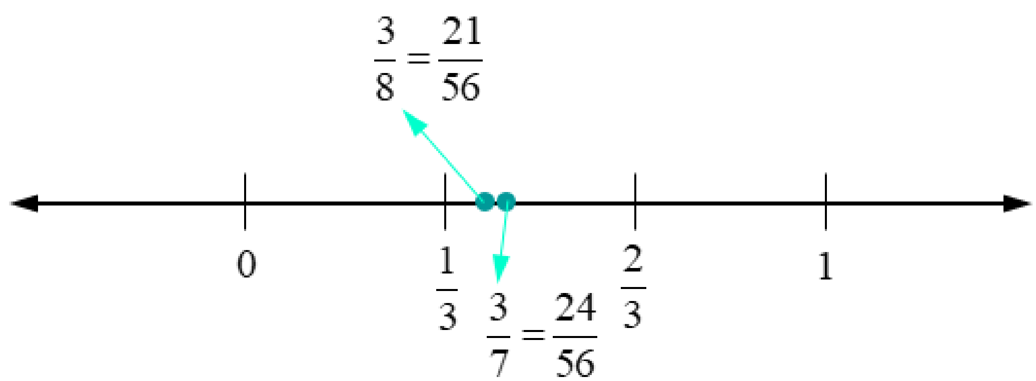
a. $\frac{2}{3} < 1$



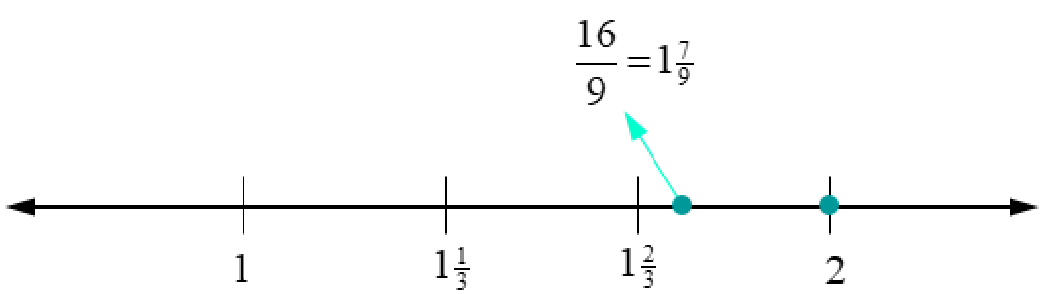
b. $3\frac{1}{2} > 3$



c. $\frac{3}{7} > \frac{3}{8}$



d. $2 > \frac{16}{9}$



Note:
Try It
Exercise:

Problem:

Order each of the following pairs of numbers, using $<$ or $>$:

$\frac{1}{3}$ __ 1
$1\frac{1}{2}$ __ 2
$\frac{2}{3}$ __ $\frac{1}{3}$
3 __ $\frac{7}{3}$

Solution:

$<$
$<$
$>$
$>$

Note:
Try It
Exercise:

Problem:

Order each of the following pairs of numbers, using $<$ or $>$:

3 __ $\frac{17}{5}$
$2\frac{1}{4}$ __ 2
$\frac{3}{5}$ __ $\frac{4}{5}$
4 __ $\frac{10}{3}$

Solution:

$<$
$>$
$<$
$>$

Summary

- **Property of One**
  - Any number, except zero, divided by itself is one.
    $\frac{a}{a} = 1$, where $a \neq 0$.
- **Mixed Numbers**
  - A **mixed number** consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$.
  - It is written as follows: $a\frac{b}{c} \qquad c \neq 0$
- **Proper and Improper Fractions**
  - The fraction a/b is a proper fraction if $a < b$ and an improper fraction if $a \geq b$.
- **Convert an improper fraction to a mixed number.**

  Divide the denominator into the numerator.
  Identify the quotient, remainder, and divisor.
  Write the mixed number as quotient $\frac{\text{remainder}}{\text{divisor}}$.
- **Convert a mixed number to an improper fraction.**

  Multiply the whole number by the denominator.
  Add the numerator to the product found in Step 1.
  Write the final sum over the original denominator.

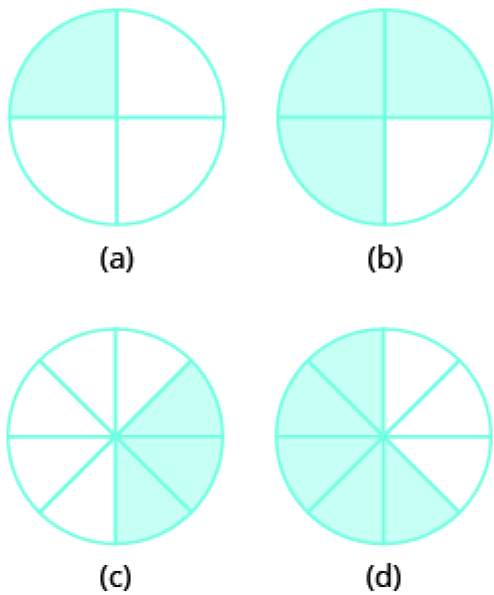
- **Equivalent Fractions Property**
  - If a, b, and c are numbers where $b \neq 0$, $c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$.

Homework

In the following exercises, name the fraction of each figure that is shaded.

Exercise:

Problem:

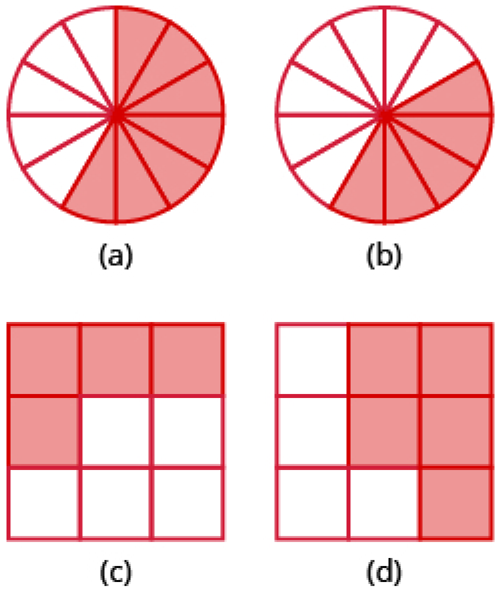


Solution:

$\frac{1}{4}$
$\frac{3}{4}$
$\frac{3}{8}$
$\frac{5}{8}$

Exercise:

Problem:

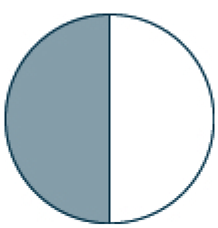


In the following exercises, shade parts of circles or squares to model the following fractions.

Exercise:

Problem: $\frac{1}{2}$

Solution:



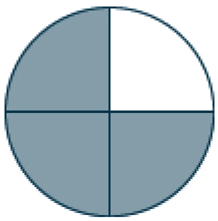
Exercise:

Problem: $\frac{1}{3}$

Exercise:

Problem: $\frac{3}{4}$

Solution:



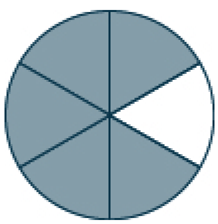
Exercise:

Problem: $\frac{2}{5}$

Exercise:

Problem: $\frac{5}{6}$

Solution:



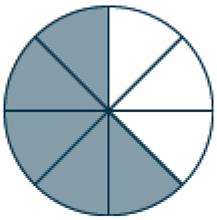
Exercise:

Problem: $\frac{7}{8}$

Exercise:

Problem: $\frac{5}{8}$

Solution:



Exercise:

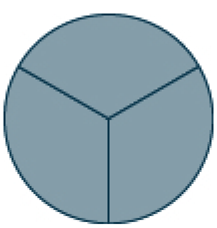
Problem: $\frac{7}{10}$

In the following exercises, use fraction circles to make wholes, if possible, with the following pieces.

Exercise:

Problem: 3 thirds

Solution:



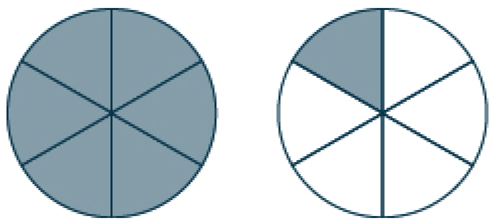
Exercise:

Problem: 8 eighths

Exercise:

Problem: 7 sixths

Solution:



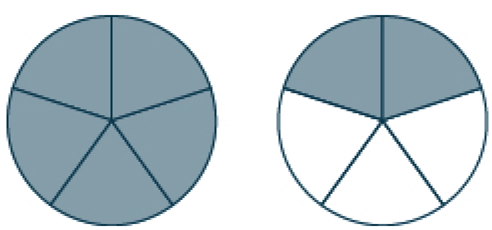
Exercise:

Problem: 4 thirds

Exercise:

Problem: 7 fifths

Solution:



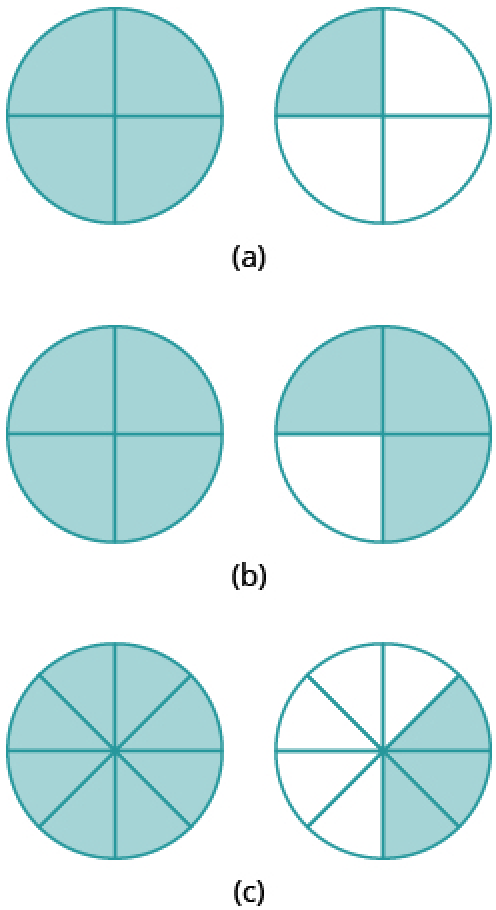
Exercise:

Problem: 7 fourths

In the following exercises, name the improper fractions. Then write each improper fraction as a mixed number.

Exercise:

Problem:

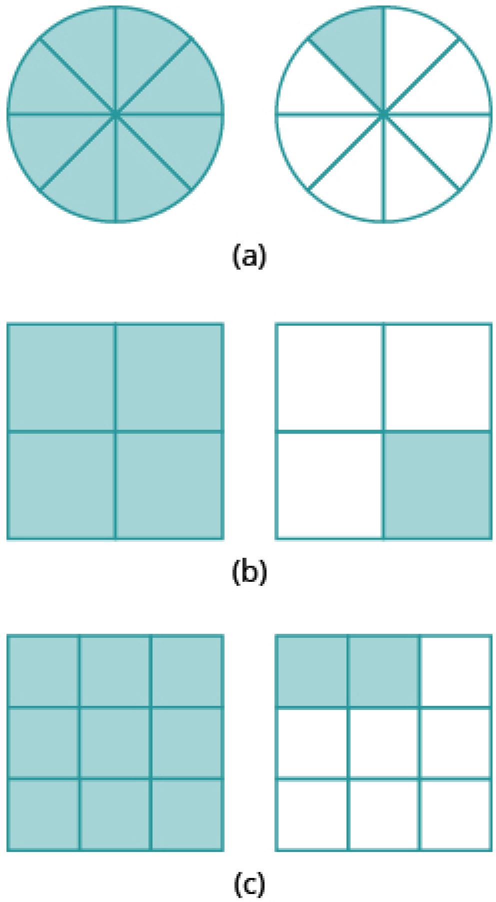


Solution:

$$\frac{5}{4} = 1\frac{1}{4}$$
$$\frac{7}{4} = 1\frac{3}{4}$$
$$\frac{11}{8} = 1\frac{3}{8}$$

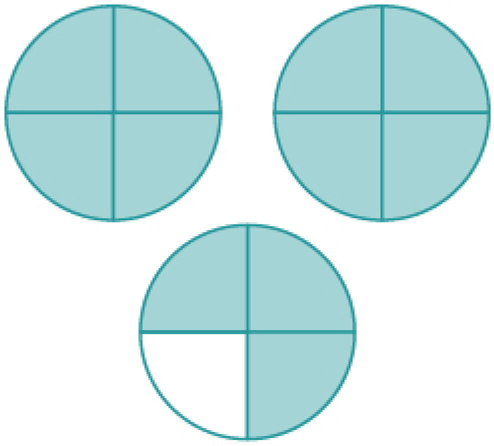
Exercise:

Problem:

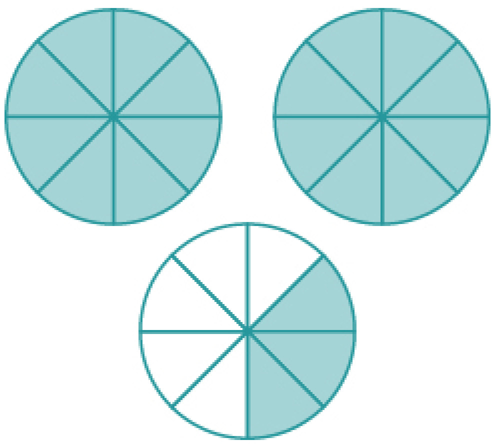


Exercise:

Problem:



(a)



(b)

Solution:

$$\frac{11}{4} = 2\frac{3}{4}$$
$$\frac{19}{8} = 2\frac{3}{8}$$

In the following exercises, draw fraction circles to model the given fraction.

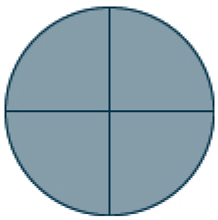
Exercise:

Problem: $\frac{3}{3}$

Exercise:

Problem: $\frac{4}{4}$

Solution:



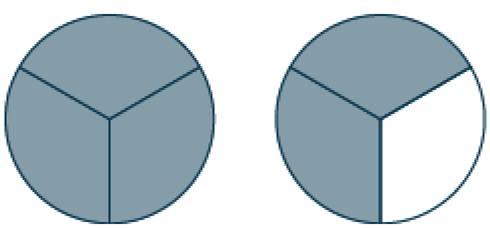
Exercise:

Problem: $\frac{7}{4}$

Exercise:

Problem: $\frac{5}{3}$

Solution:



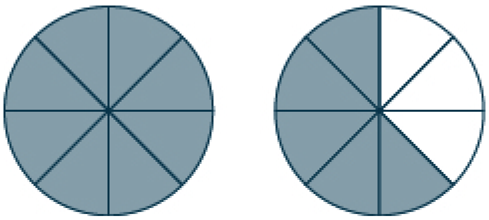
Exercise:

Problem: $\frac{11}{6}$

Exercise:

Problem: $\frac{13}{8}$

Solution:



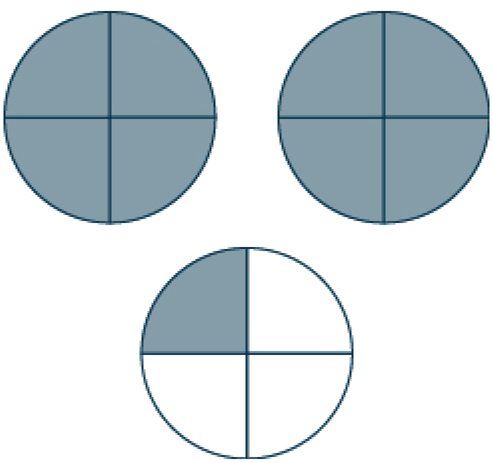
Exercise:

Problem: $\frac{10}{3}$

Exercise:

Problem: $\frac{9}{4}$

Solution:



In the following exercises, rewrite the improper fraction as a mixed number.

Exercise:

Problem: $\frac{3}{2}$

Exercise:

Problem: $\frac{5}{3}$

Solution:

$1\frac{2}{3}$

Exercise:

Problem: $\frac{11}{4}$

Exercise:

Problem: $\frac{13}{5}$

Solution:

$2\frac{3}{5}$

Exercise:

Problem: $\frac{25}{6}$

Exercise:

Problem: $\frac{28}{9}$

Solution:

$3\frac{1}{9}$

Exercise:

Problem: $\frac{42}{13}$

Exercise:

Problem: $\frac{47}{15}$

Solution:

$3\frac{2}{15}$

In the following exercises, rewrite the mixed number as an improper fraction.

Exercise:

Problem: $1\frac{2}{3}$

Exercise:

Problem: $1\frac{2}{5}$

Solution:

$\frac{7}{5}$

Exercise:

Problem: $2\frac{1}{4}$

Exercise:

Problem: $2\frac{5}{6}$

Solution:

$\frac{17}{6}$

Exercise:

Problem: $2\frac{7}{9}$

Exercise:

Problem: $2\frac{5}{7}$

Solution:

$\frac{19}{7}$

Exercise:

Problem: $3\frac{4}{7}$

Exercise:

Problem: $3\frac{5}{9}$

Solution:

$\frac{32}{9}$

In the following exercises, use fraction tiles or draw a figure to find equivalent fractions.

Exercise:

Problem: How many sixths equal one-third?

Exercise:

Problem: How many twelfths equal one-third?

Solution:

4

Exercise:

Problem: How many eighths equal three-fourths?

Exercise:

Problem: How many twelfths equal three-fourths?

Solution:

9

Exercise:

Problem: How many fourths equal three-halves?

Exercise:

Problem: How many sixths equal three-halves?

Solution:

9

In the following exercises, find three fractions equivalent to the given fraction. Show your work, using figures or algebra.

Exercise:

Problem: $\frac{1}{4}$

Exercise:

Problem: $\frac{1}{3}$

Solution:

Answers may vary. Correct answers include $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}$.

Exercise:

Problem: $\frac{3}{8}$

Exercise:

Problem: $\frac{5}{6}$

Solution:

Answers may vary. Correct answers include $\frac{10}{12}, \frac{15}{18}, \frac{20}{24}$.

Exercise:

Problem: $\frac{2}{7}$

Exercise:

Problem: $\frac{5}{9}$

Solution:

Answers may vary. Correct answers include $\frac{10}{18}, \frac{15}{27}, \frac{20}{36}$.

In the following exercises, plot the numbers on a number line.

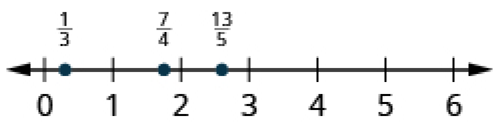
Exercise:

Problem: $\frac{2}{3}, \frac{5}{4}, \frac{12}{5}$

Exercise:

Problem: $\frac{1}{3}, \frac{7}{4}, \frac{13}{5}$

Solution:



Exercise:

Problem: $\frac{1}{4}, \frac{9}{5}, \frac{11}{3}$

Exercise:

Problem: $\frac{7}{10}, \frac{5}{2}, \frac{13}{8}, 3$

Solution:

In the following exercises, order each of the following pairs of numbers, using $<$ or $>$.

Exercise:

Problem: $1\_\_\frac{1}{4}$

Exercise:

Problem: $1\_\_\frac{1}{3}$

Solution:

$>$

Exercise:

Problem: $2\frac{1}{2}\_\_2\frac{3}{8}$

Exercise:

Problem: $1\frac{3}{4}\_\_1\frac{7}{8}$

Solution:

$<$

Exercise:

Problem: $\frac{5}{12}\_\_\frac{7}{12}$

Exercise:

Problem: $\frac{9}{10}\_\_\frac{3}{10}$

Solution:

$>$

Exercise:

Problem: $3\_\_\frac{13}{5}$

Review: Operations with Fractions

Learning Objectives

By the end of this lesson, you will be able to:

- Multiply fractions
- Divide fractions
- Add and subtract fractions

Multiplication of Fractions

Multiplication of Fractions
To multiply two fractions, multiply the numerators together and multiply the denominators together. Reduce to lowest terms if possible.

Example:
Exercise:

Problem: Multiply $\frac{3}{4} \cdot \frac{1}{6}$.

Solution:

$$\begin{aligned} \frac{3}{4} \cdot \frac{1}{6} &= \frac{3 \cdot 1}{4 \cdot 6} & \\ &= \frac{3}{24} & \text{Now reduce.} \\ &= \frac{3 \cdot 1}{2 \cdot 2 \cdot 2 \cdot 3} & \\ &= \frac{\cancel{3} \cdot 1}{2 \cdot 2 \cdot 2 \cdot \cancel{3}} & \text{3 is the only common factor.} \\ &= \frac{1}{8} & \end{aligned}$$

Notice that we since had to reduce, we nearly started over again with the original two fractions. If we factor first, then cancel, then multiply, we will save time and energy and still obtain the correct product.

Example:
Exercise:

Problem: Multiply $\frac{1}{4} \cdot \frac{8}{9}$.

Solution:

$$\begin{aligned}
\frac{1}{4} \cdot \frac{8}{9} &= \frac{1}{2 \cdot 2} \cdot \frac{2 \cdot 2 \cdot 2}{3 \cdot 3} \\
&= \frac{1}{\cancel{2} \cdot \cancel{2}} \cdot \frac{\cancel{2} \cdot \cancel{2} \cdot 2}{3 \cdot 3} \quad \text{2 is a common factor.} \\
&= \frac{1}{1} \cdot \frac{2}{3 \cdot 3} \\
&= \frac{1 \cdot 2}{1 \cdot 3 \cdot 3} \\
&= \frac{2}{9}
\end{aligned}$$

Example:
Exercise:

Problem: Multiply $\frac{3}{4} \cdot \frac{8}{9} \cdot \frac{5}{12}$.

Solution:

$$\begin{aligned}
\frac{3}{4} \cdot \frac{8}{9} \cdot \frac{5}{12} &= \frac{3}{2 \cdot 2} \cdot \frac{2 \cdot 2 \cdot 2}{3 \cdot 3} \cdot \frac{5}{2 \cdot 2 \cdot 3} \\
&= \frac{\cancel{3}}{\cancel{2} \cdot \cancel{2}} \cdot \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2}}{\cancel{3} \cdot 3} \cdot \frac{5}{\cancel{2} \cdot 2 \cdot 3} \quad \text{2 and 3 are common factors.} \\
&= \frac{1 \cdot 1 \cdot 5}{3 \cdot 2 \cdot 3} \\
&= \frac{5}{18}
\end{aligned}$$

Note:
Try It
Exercise:

Problem: Multiply $\frac{9}{14} \cdot \frac{2}{3}$.

Solution:

$\frac{3}{7}$

Division of Fractions

Reciprocals
Two numbers whose product is 1 are **reciprocals** of each other. For example, since $\frac{4}{5} \cdot \frac{5}{4} = 1$, $\frac{4}{5}$ and $\frac{5}{4}$ are reciprocals of each other. Some other pairs of reciprocals are listed below.

$$\frac{2}{7}, \frac{7}{2} \qquad \frac{3}{4}, \frac{4}{3} \qquad \frac{6}{1}, \frac{1}{6}$$

Reciprocals are used in division of fractions.

Division of Fractions
To divide a first fraction by a second fraction, multiply the first fraction by the reciprocal of the second fraction. Reduce if possible.

This method is sometimes called the "invert and multiply" method.

Perform the following divisions.

Example:
Exercise:

Problem: Divide: $\frac{3}{8} \div \frac{5}{4}$.

Solution:

$$
\begin{aligned}
&\frac{3}{8} \div \frac{5}{4}. && \text{The divisor is } \frac{5}{4}\text{. Its reciprocal is } \frac{4}{5}. \\
\frac{3}{8} \div \frac{5}{4} &= \frac{3}{8} \cdot \frac{4}{5} \\
&= \frac{3}{2 \cdot 2 \cdot 2} \cdot \frac{2 \cdot 2}{5} \\
&= \frac{3}{\cancel{2} \cdot \cancel{2} \cdot 2} \cdot \frac{\cancel{2} \cdot \cancel{2}}{5} && \text{2 is a common factor.} \\
&= \frac{3 \cdot 1}{2 \cdot 5} \\
&= \frac{3}{10}
\end{aligned}
$$

Example:
Exercise:

Problem: Divide: $\frac{5}{6} \div \frac{5}{12}$.

Solution:

$$
\begin{aligned}
&\frac{5}{6} \div \frac{5}{12}. && \text{The divisor is } \frac{5}{12}\text{. Its reciprocal is } \frac{12}{5}. \\
\frac{5}{6} \div \frac{5}{12} &= \frac{5}{6} \cdot \frac{12}{5} \\
&= \frac{5}{2 \cdot 3} \cdot \frac{2 \cdot 2 \cdot 3}{5} \\
&= \frac{\cancel{5}}{\cancel{2} \cdot \cancel{3}} \cdot \frac{\cancel{2} \cdot 2 \cdot \cancel{3}}{\cancel{5}} \\
&= \frac{1 \cdot 2}{1} \\
&= 2
\end{aligned}
$$

Note:
Try It
Exercise:

Problem: Divide: $\frac{9}{46} \div \frac{21}{23}$.

Solution:

$\frac{3}{14}$

Addition and Subtraction of Fractions

Fractions with Like Denominators
To add (or subtract) two or more fractions that have the same denominators, add (or subtract) the numerators and place the resulting sum over the common denominator. Reduce if possible.

CAUTION

Add or subtract only the numerators. Do **not** add or subtract the denominators!

Find the following sums.

Example:
Exercise:

Problem: Add: $\frac{3}{7}+\frac{2}{7}$

Solution:

$\frac{3}{7}+\frac{2}{7}.\qquad \text{The denominators are the same. Add the numerators and place the sum over 7.}$
$\frac{3}{7}+\frac{2}{7}=\frac{3+2}{7}=\frac{5}{7}$

Example:
Exercise:

Problem: Subtract: $\frac{7}{9}-\frac{4}{9}$

Solution:

$$\frac{7}{9}-\frac{4}{9}.\qquad \text{The denominators are the same.}$$
$$\frac{7}{9}-\frac{4}{9}=\frac{7-4}{9}=\frac{3}{9}=\frac{1}{3}\text{mtd}$$

Note:
Try It
Exercise:

Problem: Subtract: $\frac{5}{6}-\frac{1}{6}$.

Solution:

$\frac{2}{3}$

Fractions can only be added or subtracted conveniently if they have like denominators.

Fractions with Unlike Denominators
To add or subtract fractions having unlike denominators, convert each fraction to an equivalent fraction having as the denominator the least common multiple of the original denominators.

The least common multiple of the original denominators is commonly referred to as the **least common denominator** (LCD).

Find each sum or difference.

Example:
Exercise:

Problem: Add: $\frac{1}{6} + \frac{3}{4}$

Solution:

$$\frac{1}{6} + \frac{3}{4} \qquad \text{The denominators are not alike. Find the LCD of 6 and 4.}$$

$$\begin{cases} 6 = 2 \cdot 3 \\ 4 = 2^2 \end{cases} \qquad \text{The LCD is } 2^2 \cdot 3 = 4 \cdot 3 = 12.$$

Convert each to equivalent fractions having the common denominator 12.

$$\frac{1}{6} = \frac{1 \cdot 2}{6 \cdot 2} = \frac{2}{12} \qquad \frac{3}{4} = \frac{3 \cdot 3}{4 \cdot 3} = \frac{9}{12}$$

Now we can proceed with the addition.

$$\begin{aligned} \frac{1}{6} + \frac{3}{4} &= \frac{2}{12} + \frac{9}{12} \\ &= \frac{2+9}{12} \\ &= \frac{11}{12} \end{aligned}$$

Note:
Try It
Exercise:

Problem: Add: $\frac{1}{6}+\frac{1}{24}$.

Solution:

$\frac{5}{24}$

Example:
Exercise:

Problem: Subtract: $\frac{5}{9}-\frac{5}{12}$

Solution:

$\frac{5}{9}-\frac{5}{12}$ The denominators are not alike. Find the LCD of 9 and 12.

$\begin{cases} 9=3^2 \\ 12=2^2\cdot 3 \end{cases}$ The LCD is $2^2\cdot 3^2=4\cdot 9=36$.

Convert each to equivalent fractions having the common denominator 36.

$\frac{5}{9}=\frac{5\cdot 4}{9\cdot 4}=\frac{20}{36} \qquad \frac{5}{12}=\frac{5\cdot 3}{12\cdot 3}=\frac{15}{36}$

Now we can proceed with the subtraction.

$$\begin{aligned}\frac{5}{9}-\frac{5}{12} &= \frac{20}{36}-\frac{15}{36} \\ &= \frac{20-15}{36} \\ &= \frac{5}{36}\end{aligned}$$

Note:
Try It
Exercise:

Problem: Subtract: $\frac{13}{28}-\frac{1}{21}$.

Solution:

$\frac{5}{12}$

Homework

For the following problems, perform each indicated operation.

Exercise:

Problem: $\frac{1}{3} \cdot \frac{4}{3}$

Solution:

$\frac{4}{9}$

Exercise:

Problem: $\frac{1}{3} \cdot \frac{2}{3}$

Exercise:

Problem: $\frac{2}{5} \cdot \frac{5}{6}$

Solution:

$\frac{1}{3}$

Exercise:

Problem: $\frac{5}{6} \cdot \frac{14}{15}$

Exercise:

Problem: $\frac{9}{16} \cdot \frac{20}{27}$

Solution:

$\frac{5}{12}$

Exercise:

Problem: $\frac{35}{36} \cdot \frac{48}{55}$

Exercise:

Problem: $\frac{21}{25} \cdot \frac{15}{14}$

Solution:

$\frac{9}{10}$

Exercise:

Problem: $\frac{76}{99} \cdot \frac{66}{38}$

Exercise:

Problem: $\frac{3}{7} \cdot \frac{14}{18} \cdot \frac{6}{2}$

Solution:

1

Exercise:

Problem: $\frac{14}{15} \cdot \frac{21}{28} \cdot \frac{45}{7}$

Exercise:

Problem: $\frac{5}{9} \div \frac{5}{6}$

Solution:

$\frac{2}{3}$

Exercise:

Problem: $\frac{9}{16} \div \frac{15}{8}$

Exercise:

Problem: $\frac{4}{9} \div \frac{6}{15}$

Solution:

$\frac{10}{9}$ or $1\frac{1}{9}$

Exercise:

Problem: $\frac{25}{49} \div \frac{4}{9}$

Exercise:

Problem: $\frac{15}{4} \div \frac{27}{8}$

Solution:

$\frac{10}{9}$ or $1\frac{1}{9}$

Exercise:

Problem: $\frac{24}{75} \div \frac{8}{15}$

Exercise:

Problem: $\frac{57}{8} \div \frac{7}{8}$

Solution:

$\frac{57}{7}$ or $8\frac{1}{7}$

Exercise:

Problem: $\frac{7}{10} \div \frac{10}{7}$

Exercise:

Problem: $\frac{3}{8} + \frac{2}{8}$

Solution:

$\frac{5}{8}$

Exercise:

Problem: $\frac{3}{11} + \frac{4}{11}$

Exercise:

Problem: $\frac{5}{12} + \frac{7}{12}$

Solution:

1

Exercise:

Problem: $\frac{11}{16} - \frac{2}{16}$

Exercise:

Problem: $\frac{15}{23} - \frac{2}{23}$

Solution:

$\frac{13}{23}$

Exercise:

Problem: $\frac{3}{11} + \frac{1}{11} + \frac{5}{11}$

Exercise:

Problem: $\frac{16}{20} + \frac{1}{20} + \frac{2}{20}$

Solution:

$\frac{19}{20}$

Exercise:

Problem: $\frac{3}{8} + \frac{2}{8} - \frac{1}{8}$

Exercise:

Problem: $\frac{11}{16} + \frac{9}{16} - \frac{5}{16}$

Solution:

$\frac{15}{16}$

Exercise:

Problem: $\frac{1}{2} + \frac{1}{6}$

Exercise:

Problem: $\frac{1}{8} + \frac{1}{2}$

Solution:

$\frac{5}{8}$

Exercise:

Problem: $\frac{3}{4} + \frac{1}{3}$

Exercise:

Problem: $\frac{5}{8} + \frac{2}{3}$

Solution:

$\frac{31}{24}$ or $1\frac{7}{24}$

Exercise:

Problem: $\frac{6}{7} - \frac{1}{4}$

Exercise:

Problem: $\frac{14}{15} - \frac{1}{10}$

Solution:

$\frac{5}{6}$

Exercise:

Problem: $\frac{1}{15} + \frac{5}{12}$

Exercise:

Problem: $\frac{35}{36} - \frac{7}{10}$

Solution:

$\frac{49}{180}$

Exercise:

Problem: $\frac{9}{28} - \frac{4}{45}$

Exercise:

Problem: $\frac{8}{15} - \frac{3}{10}$

Solution:

$\frac{7}{30}$

Exercise:

Problem: $\frac{1}{16} + \frac{3}{4} - \frac{3}{8}$

Exercise:

Problem: $\frac{8}{3} - \frac{1}{4} + \frac{7}{36}$

Solution:

$\frac{47}{18}$ or $2\frac{11}{18}$

Exercise:

Problem: $\frac{3}{4} - \frac{3}{22} + \frac{5}{24}$

Review: Operations with Mixed Numbers
This is a brief review of performing the basic operations between mixed numbers.

Learning Objectives

By the end of this lesson, you will be able to:

- Multiply and divide mixed numbers.
- Add and subtract mixed numbers with common denominators.
- Add and subtract mixed numbers with different denominators.

Multiply and Divide Mixed Numbers

In the previous section, you learned how to multiply and divide fractions. All of the examples there used either proper or improper fractions. What happens when you are asked to multiply or divide mixed numbers? Remember that we can convert a mixed number to an improper fraction.

To Multiply or Divide Mixed Numbers

Convert the mixed numbers to improper fractions.
Follow the rules for fraction multiplication or division.
Simplify if possible.

Example:
Exercise:

Problem: Multiply: $3\frac{1}{3} \cdot \frac{5}{8}$

Solution:

	$3\,\frac{1}{3}\cdot\frac{5}{8}$
Convert $3\,\frac{1}{3}$ to an improper fraction.	$\frac{10}{3}\cdot\frac{5}{8}$
Multiply.	$\frac{10\cdot 5}{3\cdot 8}$
Look for common factors.	$\frac{\cancel{2}\cdot 5\cdot 5}{3\cdot\cancel{2}\cdot 4}$
Remove common factors.	$\frac{5\cdot 5}{3\cdot 4}$
Simplify.	$\frac{25}{12}$
Rewrite as a mixed number.	$2\,\frac{1}{12}$

Note:
Try It
Exercise:

Problem:

Multiply, and write your answer in simplified form. $3\,\frac{2}{5}\cdot 4\,\frac{1}{6}$.

Solution:

$14\,\frac{1}{6}$

Example:
Exercise:

Problem:

Multiply, and write your answer in simplified form: $2\frac{4}{5}\left(1\frac{7}{8}\right)$.

Solution:

	$2\frac{4}{5}\left(1\frac{7}{8}\right)$
Convert mixed numbers to improper fractions.	$\frac{14}{5}\left(\frac{15}{8}\right)$
Multiply.	$\frac{14\cdot 15}{5\cdot 8}$
Look for common factors.	$\frac{\cancel{2}\cdot 7\cdot \cancel{5}\cdot 3}{\cancel{5}\cdot \cancel{2}\cdot 4}$
Remove common factors.	$\frac{7\cdot 3}{4}$
Simplify.	$\frac{21}{4}$
Rewrite as a mixed number.	$5\frac{1}{4}$

Note:
Try It
Exercise:

Problem:

Multiply, and write your answer in simplified form: $5\frac{5}{7}\left(2\frac{5}{8}\right)$.

Solution:

15

Example:
Exercise:

Problem: Divide, and write your answer in simplified form: $3\frac{4}{7} \div 5$.

Solution:

	$3\frac{4}{7} \div 5$
Convert mixed numbers to improper fractions.	$\frac{25}{7} \div \frac{5}{1}$
Multiply the first fraction by the reciprocal of the second.	$\frac{25}{7} \cdot \frac{1}{5}$
Multiply.	$\frac{25 \cdot 1}{7 \cdot 5}$
Look for common factors.	$\frac{\cancel{5} \cdot 5 \cdot 1}{7 \cdot \cancel{5}}$
Remove common factors.	$\frac{5 \cdot 1}{7}$
Simplify.	$\frac{5}{7}$

Note:
Try It
Exercise:

Problem: Divide, and write your answer in simplified form: $4\frac{3}{8} \div 7$.

Solution:

$\frac{5}{8}$

Example:
Exercise:

Divide: $2\frac{1}{2} \div 1\frac{1}{4}$.

Problem:

Solution:

	$2\frac{1}{2} \div 1\frac{1}{4}$
Convert mixed numbers to improper fractions.	$\frac{5}{2} \div \frac{5}{4}$
Multiply the first fraction by the reciprocal of the second.	$\frac{5}{2} \cdot \frac{4}{5}$
Multiply.	$\frac{5 \cdot 4}{2 \cdot 5}$
Look for common factors.	$\frac{\cancel{5} \cdot \cancel{2} \cdot 2}{\cancel{2} \cdot 1 \cdot \cancel{5}}$
Remove common factors.	$\frac{2}{1}$
Simplify.	2

Note:
Try It
Exercise:

Problem:

Divide, and write your answer in simplified form: $3\frac{3}{4} \div 1\frac{1}{2}$.

Solution:

$2\frac{1}{2}$

Add Mixed Numbers with a Common Denominator

To Add Mixed Numbers with a Common Denominator

Add the whole numbers.
Add the fractions.
Simplify, if possible.

Example:
Exercise:

Add: $3\frac{4}{9}+2\frac{2}{9}$.

Problem:

Solution:

	$3\frac{4}{9}+2\frac{2}{9}$
Add the whole numbers.	$\begin{array}{r} 3\frac{4}{9} \\ +\,2\frac{2}{9} \\ \hline 5\phantom{\frac{4}{9}} \end{array}$
Add the fractions.	

	$$\begin{array}{r} 3\frac{4}{9} \\ +\,2\frac{2}{9} \\ \hline 5\frac{6}{9} \end{array}$$
Simplify the fraction.	$$\begin{array}{r} 3\frac{4}{9} \\ +\,2\frac{2}{9} \\ \hline 5\frac{6}{9} = 5\,\frac{2}{3} \end{array}$$

Note:
Try It
Exercise:

Problem: Find the sum: $4\frac{4}{7} + 1\frac{2}{7}$.

Solution:

$5\frac{6}{7}$

Example:
Exercise:

Find the sum: $9\frac{5}{9} + 5\frac{7}{9}$.

Problem:

Solution:

	$9\,\frac{5}{9} + 5\,\frac{7}{9}$
Add the whole numbers and then add the fractions.	$\begin{array}{r} 9\,\frac{5}{9} \\ +\,5\,\frac{7}{9} \\ \hline 14\,\frac{12}{9} \end{array}$
Rewrite $\frac{12}{9}$ as an improper fraction.	$14 + 1\,\frac{3}{9}$
Add.	$15\,\frac{3}{9}$
Simplify.	$15\,\frac{1}{3}$

Note:
Try It
Exercise:

Problem: Find the sum: $8\frac{7}{8} + 7\frac{5}{8}$.

Solution:

$16\frac{1}{2}$

An alternate method for adding mixed numbers is to convert the mixed numbers to improper fractions and then add the improper fractions. This method is usually written horizontally.

Example:
Exercise:

Problem:

Add by converting the mixed numbers to improper fractions:
$3\frac{7}{8}+4\frac{3}{8}$.

Solution:

	$3\,\frac{7}{8}+4\,\frac{3}{8}$
Convert to improper fractions.	$\frac{31}{8}+\frac{35}{8}$
Add the fractions.	$\frac{31+35}{8}$
Simplify the numerator.	$\frac{66}{8}$
Rewrite as a mixed number.	$8\,\frac{2}{8}$
Simplify the fraction.	$8\,\frac{1}{4}$

Note:
Try It
Exercise:

Problem:

Find the sum by converting the mixed numbers to improper fractions:
$5\frac{5}{9}+3\frac{7}{9}$.

Solution:

$9\frac{1}{3}$

Subtract Mixed Numbers with a Common Denominator

Note:
To Subtract Mixed Numbers with a Common Denominator

Rewrite the problem in vertical form.
Compare the two fractions.

- If the top fraction is larger than the bottom fraction, go to Step 3.
- If not, in the top mixed number, take one whole and add it to the fraction part, making a mixed number with an improper fraction.

Subtract the fractions.
Subtract the whole numbers.
Simplify, if possible.

Example:
Exercise:

Problem: Find the difference: $5\frac{3}{5} - 2\frac{4}{5}$.

Solution:

	$5\frac{3}{5}-2\frac{4}{5}$
Rewrite the problem in vertical form.	$\begin{array}{r} 5\frac{3}{5} \\ -\,2\frac{4}{5} \\ \hline \end{array}$
Since $\frac{3}{5}$ is less than $\frac{4}{5}$, take 1 from the 5 and add it to the $\frac{3}{5}$: $\left(\frac{5}{5}+\frac{3}{5}=\frac{8}{5}\right)$	$\begin{array}{r} 5\frac{3}{5} \\ -\,2\frac{4}{5} \\ \hline \end{array} \longrightarrow \begin{array}{r} 4\frac{8}{5} \\ -\,2\frac{4}{5} \\ \hline \end{array}$
Subtract the fractions.	$\begin{array}{r} 4\frac{8}{5} \\ -\,2\frac{4}{5} \\ \hline \frac{4}{5} \end{array}$
Subtract the whole parts. The result is in simplest form.	$\begin{array}{r} 4\frac{8}{5} \\ -\,2\frac{4}{5} \\ \hline 2\frac{4}{5} \end{array}$

Since the problem was given with mixed numbers, we leave the result as a mixed number.

Note:
Try It
Exercise:

Problem: Find the difference: $6\frac{4}{9}-3\frac{7}{9}$.

Solution:

$2\frac{2}{3}$

Just as we did with addition, we could subtract mixed numbers by converting them first to improper fractions. We should write the answer in the form it was given, so if we are given mixed numbers to subtract we will write the answer as a **mixed number**.

Example:
Exercise:

Problem:

Find the difference by converting to improper fractions: $9\frac{6}{11} - 7\frac{10}{11}$.

Solution:

	$9\frac{6}{11} - 7\frac{10}{11}$
Rewrite as improper fractions.	$\frac{105}{11} - \frac{87}{11}$
Subtract the numerators.	$\frac{18}{11}$
Rewrite as a mixed number.	$1\frac{7}{11}$

Note:
Try It
Exercise:

Problem:

Find the difference by converting the mixed numbers to improper fractions: $6\frac{4}{9} - 3\frac{7}{9}$.

Solution:

$2\frac{2}{3}$

Add and Subtract Mixed Numbers with Different Denominators

To add or subtract mixed numbers with different denominators, we first convert the fractions to equivalent fractions with the LCD. Then we can follow all the steps we used above for adding or subtracting fractions with like denominators.

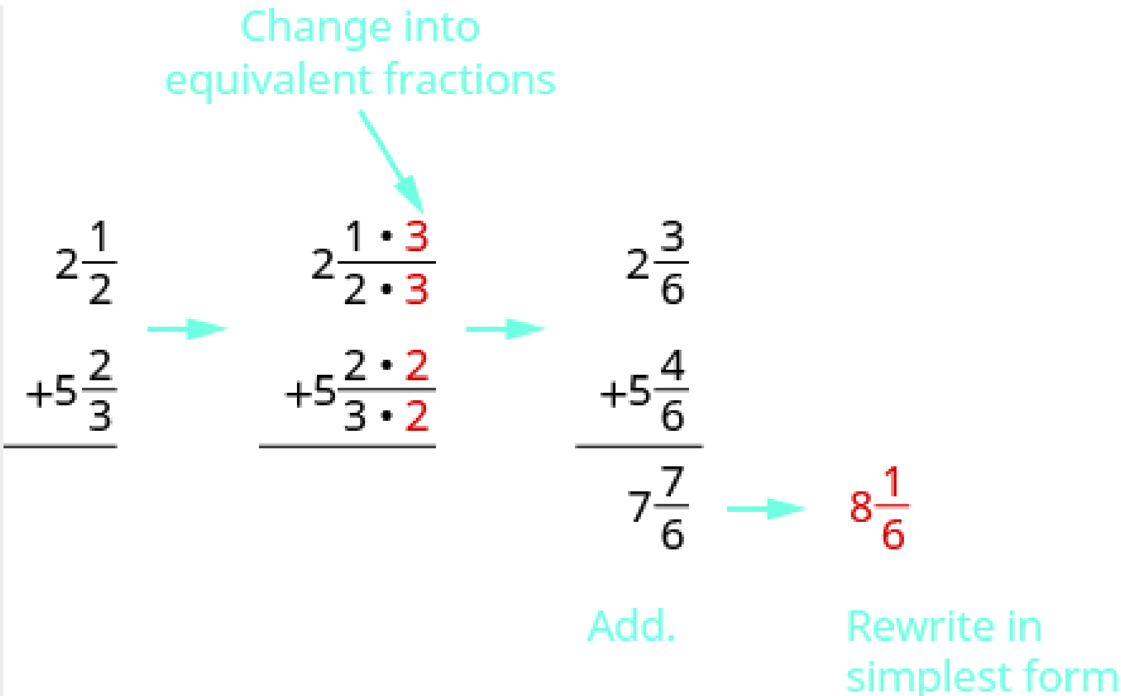
Example:
Exercise:

Add: $2\frac{1}{2} + 5\frac{2}{3}$.

Problem:

Solution:

Since the denominators are different, we rewrite the fractions as equivalent fractions with the LCD, 6. Then we will add and simplify.



We write the answer as a mixed number because we were given mixed numbers in the problem.

Note:
Try It
Exercise:

Problem: Add: $1\frac{5}{6} + 4\frac{3}{4}$.

Solution:

$6\frac{7}{12}$

Note:
Try It
Exercise:

Problem: Add: $3\frac{7}{8} + 5\frac{1}{3}$.

Solution:

$9\frac{5}{24}$

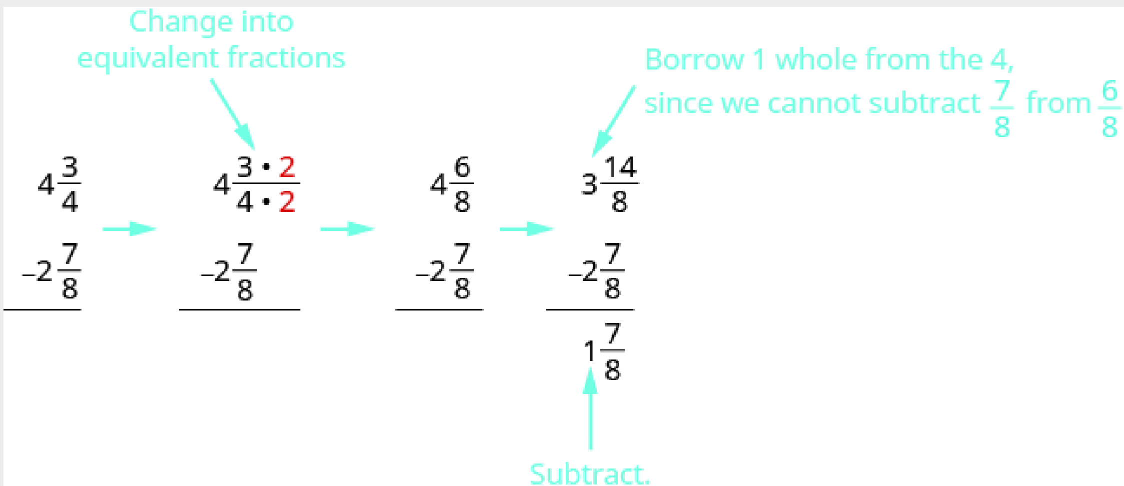
Example:
Exercise:

Subtract: $4\frac{3}{4} - 2\frac{7}{8}$.

Problem:

Solution:

Since the denominators of the fractions are different, we will rewrite them as equivalent fractions with the LCD 8. Once in that form, we will subtract. But we will need to borrow 1 first.



We were given mixed numbers, so we leave the answer as a mixed number.

Note:
Try It
Exercise:

Problem: Find the difference: $8\frac{1}{2} - 3\frac{4}{5}$.

Solution:

$4\frac{7}{10}$

Homework

Multiply and Divide Mixed Numbers

In the following exercises, multiply or divide and write the answer as a proper fraction, whole number, or mixed number, in simplified form.
Exercise:

Problem: $2\frac{4}{9} \cdot \frac{6}{7}$

Solution:

$2\frac{2}{21}$

Exercise:

Problem: $\frac{15}{22} \cdot 3\frac{3}{5}$

Exercise:

Problem: $\frac{25}{36} \cdot 6\frac{3}{10}$

Solution:

$4\frac{3}{8}$

Exercise:

Problem: $4\frac{2}{3}\left(1\frac{1}{8}\right)$

Exercise:

Problem: $2\frac{2}{5}\left(2\frac{2}{9}\right)$

Solution:

$5\frac{1}{3}$

Exercise:

Problem: $4\frac{4}{9} \cdot 5\frac{13}{16}$

Exercise:

Problem: $13\frac{1}{2} \div 9$

Solution:

$1\frac{1}{2}$

Exercise:

Problem: $12 \div 3\frac{3}{11}$

Exercise:

Problem: $7 \div 5\frac{1}{4}$

Solution:

$1\frac{1}{3}$

Exercise:

Problem: $6\frac{3}{8} \div 2\frac{1}{8}$

Exercise:

Problem: $2\frac{1}{5} \div 1\frac{1}{10}$

Solution:

2

Exercise:

Problem: $9\frac{3}{5} \div 1\frac{3}{5}$

Add Mixed Numbers with a Common Denominator

In the following exercises, add and write the answer as a proper fraction, whole number, or mixed number, in simplified form.

Exercise:

Problem: $2\frac{4}{9} + 5\frac{1}{9}$

Solution:

$7\frac{5}{9}$

Exercise:

Problem: $5\frac{1}{3} + 6\frac{1}{3}$

Exercise:

Problem: $7\frac{9}{10} + 3\frac{1}{10}$

Solution:

11

Exercise:

Problem: $4\frac{5}{8} + 9\frac{3}{8}$

Exercise:

Problem: $9\frac{2}{3} + 1\frac{2}{3}$

Solution:

$11\frac{1}{3}$

Exercise:

Problem: $3\frac{4}{5} + 6\frac{4}{5}$

Subtract Mixed Numbers with a Common Denominator

In the following exercises, subtract and write the answer as a proper fraction, whole number, or mixed number, in simplified form.

Exercise:

Problem: $2\frac{7}{12} - 1\frac{5}{12}$

Solution:

$1\frac{1}{6}$

Exercise:

Problem: $2\frac{7}{8} - 1\frac{3}{8}$

Exercise:

Problem: $19\frac{13}{15} - 13\frac{7}{15}$

Solution:

$6\frac{2}{5}$

Exercise:

Problem: $8\frac{17}{20} - 4\frac{9}{20}$

Exercise:

Problem: $5\frac{2}{9} - 3\frac{4}{9}$

Solution:

$1\frac{7}{9}$

Exercise:

Problem: $8\frac{3}{7} - 4\frac{4}{7}$

Add and Subtract Mixed Numbers with Different Denominators

In the following exercises, write the sum or difference as a proper fraction, whole number, or mixed number, in simplified form.

Exercise:

Problem: $2\frac{1}{6} + 5\frac{3}{4}$

Solution:

$7\frac{11}{12}$

Exercise:

Problem: $3\frac{1}{4}+6\frac{1}{3}$

Exercise:

Problem: $7\frac{2}{3}+8\frac{1}{2}$

Solution:

$16\frac{1}{6}$

Exercise:

Problem: $1\frac{5}{8}+4\frac{1}{2}$

Exercise:

Problem: $2\frac{5}{6}+4\frac{1}{5}$

Solution:

$7\frac{1}{30}$

Exercise:

Problem: $3\frac{2}{5}+5\frac{3}{4}$

Exercise:

Problem: $4\frac{3}{8}-3\frac{2}{3}$

Solution:

$\frac{17}{24}$

Exercise:

Problem: $6\frac{7}{8} - 2\frac{1}{3}$

Exercise:

Problem: $6\frac{5}{9} - 4\frac{2}{5}$

Solution:

$2\frac{7}{45}$

Exercise:

Problem: $9\frac{7}{10} - 2\frac{1}{3}$

Exercise:

Problem: $6\frac{4}{5} - 1\frac{1}{4}$

Solution:

$5\frac{11}{20}$

Exercise:

Problem: $4\frac{3}{4} - 1\frac{5}{6}$

Reading and Writing Decimals

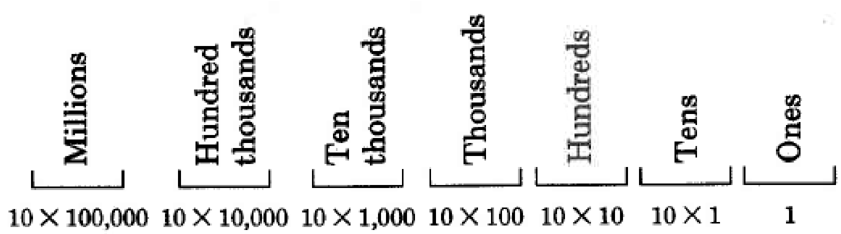
Learning Objectives

By the end of this lesson, you should be able to:

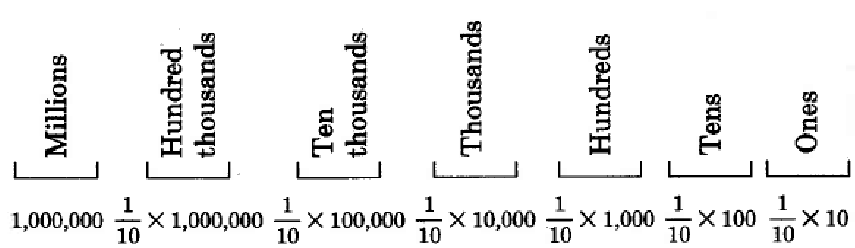
- Understand the meaning of digits to the right of the ones place
- Be familiar with the meaning of decimal fractions
- Read and write a decimal fraction.

Digits to the Right of the Ones Place

Our number system is called a positional number system with base ten. Note that each position has a particular value. Observe that each position has ten times the value of the position to its right.



This means that each position has $\frac{1}{10}$ the value of the position to its left.



Thus, a digit written to the right of the units position must have a value of $\frac{1}{10}$ of 1. Recalling that the word "of" translates to multiplication $(\cdot)$, we can see that the value of the *first position* to the right of the units digit is $\frac{1}{10}$ of 1, or

$$\frac{1}{10} \cdot 1 = \frac{1}{10}$$

The value of the *second position* to the right of the units digit is $\frac{1}{10}$ of $\frac{1}{10}$, or

$$\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10^2} = \frac{1}{100}$$

The value of the third position to the right of the units digit is $\frac{1}{10}$ of $\frac{1}{100}$, or

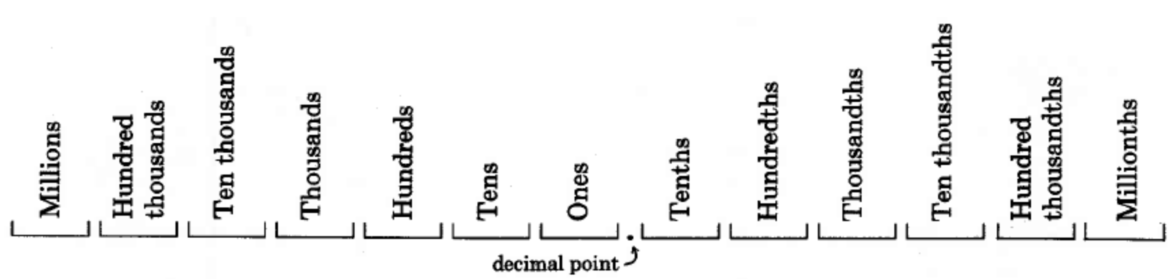
$$\frac{1}{10} \cdot \frac{1}{100} = \frac{1}{10^3} = \frac{1}{1000}$$

This pattern continues.

We can now see that if we were to write digits in positions to the right of the units positions, those positions have values that are fractions. Not only do the positions have fractional values, but the fractional values are all powers of 10 $\left(10, 10^2, 10^3, \ldots\right)$.

Decimal Fractions

Decimal Point, Decimal
If we are to write numbers with digits appearing to the right of the ones digit, we must have a way of denoting where the whole number part ends and the fractional part begins. Mathematicians denote the separation point of the ones digit and the tenths digit by writing a **decimal point**. The word *decimal* comes from the Latin prefix "deci" which means ten, and we use it because we use a base ten number system. Numbers written in this form are called **decimal fractions**, or more simply, **decimals**.



Notice that decimal numbers have the suffix "th."

Decimal Fraction

A **decimal fraction** is a fraction in which the denominator is a power of 10.

The following numbers are examples of decimals.

a. 42.6

   The 6 is in the tenths position.

   $42.6 = 42\frac{6}{10}$

b. 9.8014

   The 8 is in the tenths position.
   The 0 is in the hundredths position.
   The 1 is in the thousandths position.
   The 4 is in the ten thousandths position.

   $9.8014 = 9\frac{8014}{10{,}000}$

c. 0.93

   The 9 is in the tenths position.
   The 3 is in the hundredths position.

   $0.93 = \frac{93}{100}$

> **Note:**Quite often a zero is inserted in front of a decimal point (in the units position) of a decimal fraction that has a value less than one. This zero helps keep us from overlooking the decimal point.

d. 0.7

   The 7 is in the tenths position.

   $0.7 = \frac{7}{10}$

Note: We can insert zeros to the right of the right-most digit in a decimal fraction without changing the value of the number.
$\frac{7}{10} = 0.7 = 0.70 = \frac{70}{100} = \frac{7}{10}$

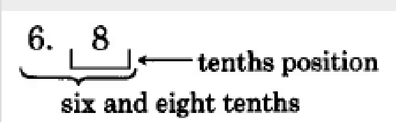
Reading Decimal Fractions

Reading a Decimal Fraction
To read a decimal fraction,

1. Read the whole number part as usual. (If the whole number is less than 1, omit steps 1 and 2.)
2. Read the decimal point as the word "and."
3. Read the number to the right of the decimal point as if it were a whole number.
4. Say the name of the place value of the last digit.

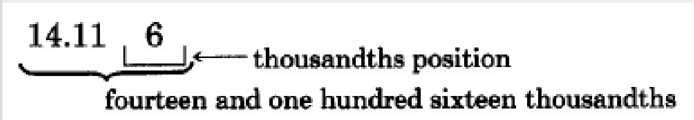
Read the following numbers.

Example:
6.8

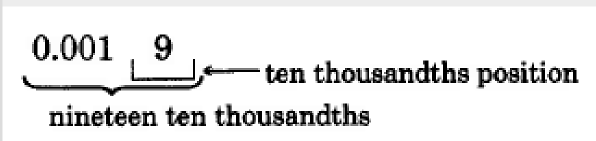


Note: Some people read this as "six point eight." This phrasing gets the message across, but technically, "six *and* eight tenths" is the correct phrasing.

Example:
14.116



Example:
0.0019



Example:
81
Eighty-one
In this problem, the indication is that any whole number is a decimal fraction. Whole numbers are often called *decimal numbers*.
$81 = 81.0$

Practice Exercises

Read the following numbers.

Note:
Try It
Exercise:

Problem: 12.9

Solution:

twelve and nine tenths

Note:
Try It
Exercise:

Problem: 4.86

Solution:

four and eighty-six hundredths

Note:
Try It
Exercise:

Problem: 7.00002

Solution:

seven and two hundred-thousandths

Writing Decimal Fractions

Writing a Decimal Fraction

To write a decimal fraction,

1. Write the whole number part.
2. Write a decimal point for the word "and."
3. Write the decimal part of the number so that the right-most digit appears in the place value indicated in the word name. If necessary, insert zeros to the right of the decimal point in order that the right-most digit appears in the correct position.

Write each number.

Example:
Thirty-one and twelve hundredths.
The decimal position indicated is the hundredths position.
31.12

Example:
Two and three hundred-thousandths.
The decimal position indicated is the hundred thousandths. We'll need to insert enough zeros to the immediate right of the decimal point in order to locate the 3 in the correct position.
2.00003

Example:
Six thousand twenty-seven and one hundred four millionths.
The decimal position indicated is the millionths position. We'll need to insert enough zeros to the immediate right of the decimal point in order to locate the 4 in the correct position.
6,027.000104

Practice Exercises

Write each number.

Note:
Try It
Exercise:

Problem: Three hundred six and forty-nine hundredths.

Solution:

306.49

Note:
Try It
Exercise:

Problem: Nine and four thousandths.

Solution:

9.004

Note:
Try It
Exercise:

Problem: Sixty-one millionths.

Solution:

0.000061

Homework

For the following three problems, give the decimal name of the position of the given number in each decimal fraction.

Exercise:

Problem:
1. 3.941
9 is in the position.
4 is in the position.
1 is in the position.

Solution:

Tenths; hundredths, thousandths

Exercise:

Problem:
17.1085
1 is in the position.
0 is in the position.
8 is in the position.
5 is in the position.

Exercise:

Problem:
652.3561927
9 is in the position.
7 is in the position.

Solution:

Hundred thousandths; ten millionths

For the following 7 problems, read each decimal fraction by writing it.

Exercise:

Problem: 9.2

Exercise:

Problem: 8.1

Solution:

eight and one tenth

Exercise:

Problem: 10.15

Exercise:

Problem: 55.06

Solution:

fifty-five and six hundredths

Exercise:

Problem: 0.78

Exercise:

Problem: 1.904

Solution:

one and nine hundred four thousandths

Exercise:

Problem: 10.00011

For the following 10 problems, write each decimal fraction.

Exercise:

Problem: Three and twenty-one hundredths.

Solution:

3.21

Exercise:

Problem: Fourteen and sixty-seven hundredths.

Exercise:

Problem: One and eight tenths.

Solution:

1.8

Exercise:

Problem: Sixty-one and five tenths.

Exercise:

Problem: Five hundred eleven and four thousandths.

Solution:

511.004

Exercise:

Problem: Thirty-three and twelve ten-thousandths.

Exercise:

Problem: Nine hundred forty-seven thousandths.

Solution:

0.947

Exercise:

Problem: Two millionths.

Exercise:

Problem: Seventy-one hundred-thousandths.

Solution:

0.00071

Exercise:

Problem: One and ten ten-millionths.

Calculator Problems

For the following 10 problems, perform each division using a calculator. Then write the resulting decimal using words.

Exercise:

Problem: $3 \div 4$

Solution:

seventy-five hundredths

Exercise:

Problem: $1 \div 8$

Exercise:

Problem: $4 \div 10$

Solution:

four tenths

Exercise:

Problem: $2 \div 5$

Exercise:

Problem: $4 \div 25$

Solution:

sixteen hundredths

Exercise:

Problem: $1 \div 50$

Exercise:

Problem: $3 \div 16$

Solution:

one thousand eight hundred seventy-five ten thousandths

Exercise:

Problem: $15 \div 8$

Exercise:

Problem: $11 \div 20$

Solution:

fifty-five hundredths

Exercise:

Problem: $9 \div 40$

Rounding Decimals
This module discusses how to round decimals.

Learning Objectives

By the end of this lesson, you should be able to:

- Round a decimal number to a specified place value
- Round a decimal number to a specified number of decimal places

Rounding Decimal Numbers

We first considered the concept of rounding numbers in [link] where our concern with rounding was related to whole numbers only. With a few minor changes, we can apply the same rules of rounding to decimals.

To round a decimal to a particular position:

1. Mark the position of the round-off digit (with an arrow or check).
2. Note whether the digit to the immediate right of the marked digit is

   a. *less than 5*. If so, leave the round-off digit unchanged.
   b. *5 or greater*. If so, add 1 to the round-off digit.

3. If the round-off digit is

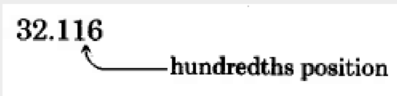
   a. to the right of the decimal point, eliminate all the digits to its right.
   b. to the left of the decimal point, replace all the digits between it and the decimal point with zeros and eliminate the decimal point and all the decimal digits.

Examples

Round each decimal to the specified place value. (The numbers in parentheses indicate which step is being used.)

Example:
Round 32.116 to the nearest hundredth.

- **1**

- **2b** The digit immediately to the right is 6, and 6 > 5, so we add 1 to the round-off digit:

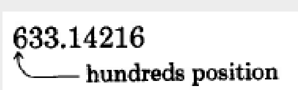
  $1 + 1 = 2$
- **3a** The round-off digit is to the right of the decimal point, so we eliminate all digits to its right.

  32.12

The number 32.116 rounded to the nearest hundredth is 32.12.

Example:
Round 633.14216 to the nearest hundred.

- **1**

- **2a** The digit immediately to the right is 3, and 3 < 5 so we leave the round-off digit unchanged.
- **3b** The round-off digit is to the left of 0, so we replace all the digits between it and the decimal point with zeros and eliminate the decimal point and all the decimal digits.

  600

The number 633.14216 rounded to the nearest hundred is 600.

Example:
1,729.63 rounded to the nearest ten is 1,730.

Example:
1.0144 rounded to the nearest tenth is 1.0.

Example:
60.98 rounded to the nearest one is 61.

Sometimes we hear a phrase such as "round to three decimal places." This phrase means that the round-off digit is the third decimal digit (the digit in the thousandths position).

Example:
67.129 rounded to the second decimal place is 67.13.

Example:
67.129558 rounded to 3 decimal places is 67.130.

Practice

Round each decimal to the specified place value or number of decimal places.

Note:

Try It
Exercise:

Problem: 4.816 to the nearest hundredth.

Solution:

4.82

Note:
Try It
Exercise:

Problem: 0.35928 to the nearest ten thousandths.

Solution:

0.3593

Note:
Try It
Exercise:

Problem: 82.1 to the nearest one.

Solution:

82

Note:

Try It
Exercise:

Problem: 753.98 to the nearest hundred.

Solution:

800

Note:
Try It
Exercise:

Problem: Round 43.99446 to three decimal places.

Solution:

43.994

Note:
Try It
Exercise:

Problem: Round 105.019997 to four decimal places.

Solution:

105.0200

Homework

For the first 10 problems, complete the chart by rounding each decimal to the indicated place value.

Exercise:

Problem: 20.01071

Tenth	Hundredth	Thousandth	Ten Thousandth

Solution:

Tenth	Hundredth	Thousandth	Ten Thousandth
20.0	20.01	20.011	20.0107

Exercise:

Problem: 3.52612

Tenth	Hundredth	Thousandth	Ten Thousandth
	3.53		

Exercise:

Problem: 531.21878

Tenth	Hundredth	Thousandth	Ten Thousandth

Solution:

Tenth	Hundredth	Thousandth	Ten Thousandth
531.2	531.22	531.219	531.2188

Exercise:

Problem: 36.109053

Tenth	Hundredth	Thousandth	Ten Thousandth
36.1			

Exercise:

Problem: 1.999994

Tenth	Hundredth	Thousandth	Ten Thousandth

Solution:

Tenth	Hundredth	Thousandth	Ten Thousandth
2.0	2.00	2.000	2.0000

Exercise:

Problem: 7.4141998

Tenth	Hundredth	Thousandth	Ten Thousandth
		7.414	

Exercise:

Problem: 0.000007

Tenth	Hundredth	Thousandth	Ten Thousandth

Solution:

Tenth	Hundredth	Thousandth	Ten Thousandth
0.0	0.00	0.000	0.0000

Exercise:

Problem: 0.00008

Tenth	Hundredth	Thousandth	Ten Thousandth
			0.0001

Exercise:

Problem: 9.19191919

Tenth	Hundredth	Thousandth	Ten Thousandth

Solution:

Tenth	Hundredth	Thousandth	Ten Thousandth
9.2	9.19	9.192	9.1919

Exercise:

Problem: 0.0876543

Tenth	Hundredth	Thousandth	Ten Thousandth

Calculator Problems

For the following 5 problems, round 18.4168095 to the indicated number of decimal places.

Exercise:

Problem: 3 decimal places.

Solution:

18.417

Exercise:

Problem: 1 decimal place.

Exercise:

Problem: 5 decimal places.

Solution:

18.41681

Exercise:

Problem: 6 decimal places.

Exercise:

Problem: 2 decimal places.

Solution:

18.42

Calculator Problems

For the following problems, perform each division using a calculator.

Exercise:

Problem: $4 \div 3$ and round to 2 decimal places.

Exercise:

Problem: $1 \div 8$ and round to 1 decimal place.

Solution:

0.1

Exercise:

Problem: $1 \div 27$ and round to 6 decimal places.

Exercise:

Problem: $51 \div 61$ and round to 5 decimal places.

Solution:

0.83607

Exercise:

Problem: $3 \div 16$ and round to 3 decimal places.

Exercise:

Problem: $16 \div 3$ and round to 3 decimal places.

Solution:

5.333

Exercise:

Problem: $26 \div 7$ and round to 5 decimal places.

Multiplying and Dividing by Powers of Ten
In this module, students will learn how to multiply and divide by powers of ten mentally by shifting the decimal.

Powers of Ten
Powers of ten are numbers that result from raising 10 to an integer exponent. These numbers are easy to identify. They have a single digit of 1 and all other digits are zeros.

Examples of powers of ten include: 1000, 100, 10, 0.1, 0.01, and 0.001.

In many situations, you'll want to be able to multiply and divide by 1000, 100, 10, 0.1, 0.01, 0.001 etc. **without a calculator**, which can be done by looking at the number and deciding which direction to move the decimal point and how many places.

Which direction do we move the decimal point?
We expect to get either a larger or smaller result based on the operation and the base ten being used.

You will need to keep in mind that:

Moving the decimal **left** results in a **smaller** number.

Moving the decimal **right** results in a **larger** number.

How many places do we move the decimal?
How many places we move the decimal will depend on which power of ten we are multiplying or dividing by.

The key is to count the number of places the 1 digit is away from the ones place, in the power of ten you are multiplying or dividing by.

Multiplication

When multiplying a number by a whole number greater than 1 (like 1000, 100, 10 etc.) the result is a larger number, which means the decimal on the

other number needs to be moved to the right.

For example, if we are multiplying by 1000, then since the 1 in 1000 is three places away from the ones place, we move the decimal 3 places and since we need to get a larger number as a result, we move the decimal to the right.

Example:
Exercise:

Problem: Multiply: 46,500(1000)

Solution:

Since we are multiplying by 1000, we move the decimal on the number 46,500 three places to the right. This will leave three blank spaces, which will need to filled in with zeros as place holders.

Thus, 46,500(1000) = 46,500,000.

Example:
Exercise:

Problem: Multiply 382.74 × 10

Solution:

Since the 1 in 10 is one place away from the ones place, the decimal must be moved one place. The number 10 is a whole number, so the result must be larger, which means the decimal must move to the right.

Moving the decimal on 382.74 one place to the right gives us 3827.4.

Note:
Try It
Exercise:

Problem: Multiply: 375(10,000)

Solution:

3,750,000

Multiplying by a fraction or decimal number less than one (like 0.1, 0.01, 0.001 etc.), creates a smaller number, which means the decimal in the other number needs to be moved to the left.

For example, if we are multiplying by 0.0001, then since the 1 in 0.0001 is four places away from the ones place, we move the decimal 4 places and since we need to get a smaller number as a result, we move the decimal to the left.

Example:
Exercise:

Problem: Multiply: 98,650 × 0.01

Solution:

Since 0.01 is a fraction or decimal (in other words, a number less than 1) the result will be smaller than 98,650, which means the decimal will move to the left. The digit of 1 in the number 0.01 is two places from the ones place, so the decimal needs to be moved two places to

the left.

Moving the decimal on 98,650 two places to the left gives us 986.5.

Note:
Try It
Exercise:

Problem: Multiply 52.34(0.001).

Solution:

0.05234

Division

Division is the opposite of multiplication. So, as you would expect, we get the opposite results.

When dividing by a whole number greater than 1 (like 1000, 100, 10, etc.), the result is smaller than the number being divided.

When dividing by a fraction or decimal less than 1 (like 0.1, 0.01, 0.001, etc.), the result will be larger than the number being divided.

Example:
Exercise:

Problem: Divide: 46,500 ÷ 1000

Solution:

Since we are dividing by a whole number greater than 1, namely 1000, the result will be smaller than 46,500. So, the decimal needs to be moved to the left. The 1 in the number 1000 is three places away from the ones place, so the decimal should be moved three places to the left.

Moving the decimal in the number 46,500 three places to the left gives us 46.5.

Note:
Try It
Exercise:

Problem: Divide: 8,756.374 ÷ 10,000

Solution:

0.8756374

Example:
Exercise:

Problem: Divide: 98,650 ÷ 0.01

Solution:

Since 0.01 is a decimal number less than one, the result of this division will be larger than 98,650. Thus, the decimal needs to move to the right. The 1 in the number 0.01 is two places away from the ones place, so the decimal moves two places to the right.

The result of moving the decimal in the number 98,650 to the right two places is 9,865,000.

Note:
Try It
Exercise:

Problem: Divide: $465.18 \div 0.0001$

Solution:

4,651,800

Homework

Complete the following problems without a calculator.
Exercise:

Problem: Divide: $8{,}457.3 \div 100$

Solution:

84.573

Exercise:

Problem: Multiply: $60{,}048 \times 0.001$

Exercise:

Problem: Multiply: $10{,}000(9.3)$

Solution:

93,000

Exercise:

Problem: Divide: 5 ÷ 100

Exercise:

Problem: Divide: $\frac{435,091.3}{100,000}$

Solution:

4.350913

Exercise:

Problem: Multiply: 0.00425 × 0.001

Exercise:

Problem: Multiply: 758(1,000,000)

Solution:

758,000,000

Exercise:

Problem: Divide: 92.475 ÷ 0.0001

Exercise:

Problem: Divide: 0.54 ÷ 0.0001

Solution:

5,400

Exercise:

Problem: Multiply: $72.4 \times 10{,}000$

Ratios and Rates

Learning Objectives

By the end of this section, you will be able to:

- Write a ratio as a fraction
- Write a rate as a fraction
- Find unit rates
- Find unit price
- Translate phrases to expressions with fractions

Write a Ratio as a Fraction

When you apply for a mortgage, the loan officer will compare your total debt to your total income to decide if you qualify for the loan. This comparison is called the debt-to-income ratio. A **ratio** compares two quantities that are measured with the same unit. If we compare a and b, the ratio is written as $\frac{a}{b}$ or $a{:}b$.

Note:
Ratios
A **ratio** compares two numbers or two quantities that are measured with the same unit. The ratio of a to b is written as $\frac{a}{b}$ or $a{:}b$. **In mathematics, writing a ratio in fraction form is preferred.**

In this section, we will use the fraction notation. When a ratio is written in fraction form, the fraction should be simplified. If it is an improper fraction, we do not change it to a mixed number. Because a ratio compares two quantities, we would leave a ratio as $\frac{4}{1}$ instead of simplifying it to 4 so that we can see the two parts of the ratio.

Example:
Exercise:

Problem: Write each ratio as a fraction:

a. 15 to 27
b. 45 to 18

Solution:

a	
	15 to 27
Write as a fraction with the first number in the numerator and the second in the denominator.	$\frac{15}{27}$
Simplify the fraction.	$\frac{5}{9}$

b	
	45 to 18
Write as a fraction with the first number in the numerator and the second in the denominator.	$\frac{45}{18}$
Simplify.	$\frac{5}{2}$

We leave the ratio in b as an improper fraction. (We NEVER write a ratio as a mixed number.)

Note:
Try It
Exercise:

Problem: Write each ratio as a fraction:

a. 21 to 56
b. 48 to 32

Solution:

$\frac{3}{8}$
$\frac{3}{2}$

Note:
Try It
Exercise:

Problem: Write each ratio as a fraction:

a. 27 to 72
b. 51 to 34

Solution:

$\frac{3}{8}$
$\frac{3}{2}$

Ratios Involving Decimals

We will often work with ratios of decimals, especially when we have ratios involving money. In these cases, we can eliminate the decimals by using the Equivalent Fractions Property to convert the ratio to a fraction with whole numbers in the numerator and denominator.

For example, consider the ratio 0.8 to 0.05. We can write it as a fraction with decimals and then multiply the numerator and denominator by 100 to eliminate the decimals.

$$\frac{0.8}{0.05}$$

$$\frac{(0.8)100}{(0.05)100}$$

$$\frac{80}{5}$$

Do you see a shortcut to find the equivalent fraction? Notice that $0.8 = \frac{8}{10}$ and $0.05 = \frac{5}{100}$. The least common denominator of $\frac{8}{10}$ and $\frac{5}{100}$ is 100. By multiplying the numerator and denominator of $\frac{0.8}{0.05}$ by 100, we 'moved' the decimal two places to the right to get the equivalent fraction with no decimals. Now that we understand the math behind the process, we can find the fraction with no decimals like this:

	$\frac{0.80}{0.05}$
"Move" the decimal 2 places.	$\frac{80}{5}$
Simplify.	$\frac{16}{1}$

You do not have to write out every step when you multiply the numerator and denominator by powers of ten. As long as you move both decimal places the same number of places, the ratio will remain the same.

Example:
Exercise:

Problem: Write each ratio as a fraction of whole numbers:

a. 4.8 to 11.2

b. 2.7 to 0.54

Solution:

a. 4.8 to 11.2	
Write as a fraction.	$\frac{4.8}{11.2}$
Rewrite as an equivalent fraction without decimals, by moving both decimal points 1 place to the right.	$\frac{48}{112}$
Simplify.	$\frac{3}{7}$

So 4.8 to 11.2 is equivalent to $\frac{3}{7}$.

b.
The numerator has one decimal place and the denominator has 2. To clear both decimals we need to move the decimal 2 places to the right.

2.7 to 0.54

Write as a fraction.	$\frac{2.7}{0.54}$
Move both decimals right two places.	$\frac{270}{54}$
Simplify.	$\frac{5}{1}$

So 2.7 to 0.54 is equivalent to $\frac{5}{1}$.

Note:

Try It
Exercise:

Problem: Write each ratio as a fraction:

a. 4.6 to 11.5
b. 2.3 to 0.69

Solution:

$\frac{2}{5}$
$\frac{10}{3}$

Note:
Try It
Exercise:

Problem: Write each ratio as a fraction:

a. 3.4 to 15.3
b. 3.4 to 0.68

Solution:

$\frac{2}{9}$
$\frac{5}{1}$

Some ratios compare two mixed numbers. Remember that to divide mixed numbers, you first rewrite them as improper fractions.

Example:
Exercise:

Problem: Write the ratio of $1\frac{1}{4}$ to $2\frac{3}{8}$ as a fraction.

Solution:

	$1\frac{1}{4}$ to $2\frac{3}{8}$
Write as a fraction.	$\frac{1\frac{1}{4}}{2\frac{3}{8}}$
Convert the numerator and denominator to improper fractions.	$\frac{\frac{5}{4}}{\frac{19}{8}}$
Rewrite as a division of fractions.	$\frac{5}{4} \div \frac{19}{8}$
Invert the divisor and multiply.	$\frac{5}{4} \cdot \frac{8}{19}$
Simplify.	$\frac{10}{19}$

Note:
Try It
Exercise:

Problem: Write each ratio as a fraction: $1\frac{3}{4}$ to $2\frac{5}{8}$.

Solution:

$\frac{2}{3}$

Note:
Try It

Exercise:

Problem: Write each ratio as a fraction: $1\frac{1}{8}$ to $2\frac{3}{4}$.

Solution:

$\frac{9}{22}$

Applications of Ratios

HDL Cholesterol
One real-world application of ratios that affects many people involves measuring cholesterol in blood. The ratio of total cholesterol to HDL cholesterol is one way doctors assess a person's overall health. A ratio of less than 5 to 1 is considered good.

Example:
Exercise:

Problem:

Hector's total cholesterol is 249 mg/dL and his HDL cholesterol is 39 mg/dL.

a. Find the ratio of his total cholesterol to his HDL cholesterol.
b. Assuming that a ratio less than 5 to 1 is considered good, what would you suggest to Hector?

Solution:

a. First, write the words that express the ratio. We want to know the ratio of Hector's total cholesterol to his HDL cholesterol.

Write as a fraction.	$\frac{\text{total cholesterol}}{\text{HDL cholesterol}}$
Substitute the values.	$\frac{249}{39}$

Simplify.	$\frac{83}{13}$

b. Is Hector's cholesterol ratio ok? If we divide 83 by 13 we obtain approximately 6.4, so $\frac{83}{13} \approx \frac{6.4}{1}$. Hector's cholesterol ratio is high! Hector should either lower his total cholesterol or raise his HDL cholesterol.

Note:
Try It
Exercise:

Problem:

Find the patient's ratio of total cholesterol to HDL cholesterol using the given information.

Total cholesterol is 185 mg/dL and HDL cholesterol is 40 mg/dL.

Solution:

$\frac{37}{8}$

Note:
Try It
Exercise:

Problem:

Find the patient’s ratio of total cholesterol to HDL cholesterol using the given information.

Total cholesterol is 204 mg/dL and HDL cholesterol is 38 mg/dL.

Solution:

$\frac{102}{19}$

Chemical-to-Solution Ratio

Ratios are often used to show the concentration of a solution. This type of ratio is called a **chemical-to-solution ratio** and is defined as the ratio of the amount of chemical to the total amount of solution (chemical plus water or solvent). Notice how a ratio of a chemical to a solution is calculated:

Equation:

$$\frac{\text{amount of chemical}}{\text{total amount of solution}} = \frac{\text{amount of chemical}}{\text{amount of chemical} + \text{amount of water (or solvent)}}$$

It is common to reduce this ratio so the denominator is a one (by dividing the numerator by the denominator) in order to compare solution concentrations.

Example:

Exercise:

Problem:

Given 35 oz. of a chemical combined with 65 oz. of water, write the chemical-to-solution ratio and reduce so the denominator is one.

Solution:

The amount of chemical needs to be placed in the numerator, while the sum of the amount of chemical and the amount of water needs to be placed in the denominator. Then, simplify.

Equation:

$$\frac{35 \text{ oz.}}{35 \text{ oz.} + 65 \text{ oz.}} = \frac{35 \text{ oz.}}{100 \text{ oz.}}$$

Now, we divide to reduce the denominator to one.

Equation:

$$\frac{35 \text{ oz.}}{100 \text{ oz.}} = \frac{0.35}{1}$$

This solution has a concentration of 0.35 to 1.

Note:
Try It
Exercise:

Problem:

Given 50 oz. of a chemical combined with 150 oz. of water, write the chemical-to-solution ratio and reduce so the denominator is one.

Solution:

$\frac{0.25}{1}$

Ratios of Two Measurements in Different Units

To find the ratio of two measurements, we must make sure the quantities have been measured with the same unit. If the measurements are not in the same units, we must first convert them to the same units.

We know that to simplify a fraction, we divide out common factors. Similarly in a ratio of measurements, we divide out the common unit.

Example:
Exercise:

Problem:

The Americans with Disabilities Act (ADA) Guidelines for wheel chair ramps require a maximum vertical rise of 1 inch for every 1 foot of horizontal run. What is the ratio of the rise to the run?

Solution:

In a ratio, the measurements must be in the same units. We can change feet to inches, or inches to feet. It is usually easier to convert to the smaller unit, since this avoids introducing more fractions into the problem.

Write the words that express the ratio.

	Ratio of the rise to the run
Write the ratio as a fraction.	$\frac{\text{rise}}{\text{run}}$
Substitute in the given values.	$\frac{1\text{ inch}}{1\text{ foot}}$
Convert 1 foot to inches.	$\frac{1\text{ inch}}{12\text{ inches}}$
Simplify, dividing out common factors and units.	$\frac{1}{12}$

So the ratio of rise to run is 1 to 12. This means that the ramp should rise 1 inch for every 12 inches of horizontal run to comply with the guidelines.

Note:
Try It
Exercise:

Problem: Find the ratio of the first length to the second length: 1 foot to 54 inches.

Solution:

$\frac{2}{9}$

Write a Rate as a Fraction

Frequently we want to compare two different types of measurements, such as miles to gallons. To make this comparison, we use a **rate**. Examples of rates are 120 miles in 2 hours, 160 words in 4 minutes, and $5 dollars per 64 ounces.

Note:
Rate
A **rate** compares two quantities of different units.

When writing a fraction as a rate, we put the first given amount with its units in the numerator and the second amount with its units in the denominator. When rates are simplified, the units remain in the numerator and denominator.

Example:
Exercise:

Problem: Bob drove his car 525 miles in 9 hours. Write this rate as a fraction.

Solution:

	525 miles in 9 hours
Write as a fraction, with 525 miles in the numerator and 9 hours in the denominator.	$\frac{525\text{ miles}}{9\text{ hours}}$
	$\frac{175\text{ miles}}{3\text{ hours}}$

So 525 miles in 9 hours is equivalent to $\frac{175\text{ miles}}{3\text{ hours}}$.

Note:
Try It
Exercise:

Problem: Write the rate as a fraction: 492 miles in 8 hours.

Solution:

$\frac{123\text{ miles}}{2\text{ hours}}$

Note:

Try It
Exercise:

Problem: Write the rate as a fraction: 242 miles in 6 hours.

Solution:

$\frac{121 \text{ miles}}{3 \text{ hours}}$

Find Unit Rates

In the last example, we calculated that Bob was driving at a rate of $\frac{175 \text{ miles}}{3 \text{ hours}}$. This tells us that every three hours, Bob will travel 175 miles. This is correct, but not very useful. We usually want the rate to reflect the number of miles in one hour. A rate that has a denominator of 1 unit is referred to as a **unit rate**.

Note:
Unit Rate
A **unit rate** is a rate with denominator of 1 unit.

Unit rates are very common in our lives. For example, when we say that we are driving at a speed of 68 miles per hour we mean that we travel 68 miles in 1 hour. We would write this rate as 68 miles/hour (read 68 miles per hour). The common abbreviation for this is 68 mph. Note that when no number is written before a unit, it is assumed to be 1.

So 68 miles/hour really means $68 \text{ miles}/1 \text{ hour}$.

Two rates we often use when driving can be written in different forms, as shown:

Example	Rate	Write	Abbreviate	Read

Example	Rate	Write	Abbreviate	Read
68 miles in 1 hour	$\frac{68\text{ miles}}{1\text{ hour}}$	68 miles/hour	68 mph	68 miles per hour
36 miles to 1 gallon	$\frac{36\text{ miles}}{1\text{ gallon}}$	36 miles/gallon	36 mpg	36 miles per gallon

Another example of unit rate that you may already know about is hourly pay rate. It is usually expressed as the amount of money earned for one hour of work. For example, if you are paid $\$12.50$ for each hour you work, you could write that your hourly (unit) pay rate is $\$12.50/\text{hour}$ (read $\$12.50$ per hour.)

To convert a rate to a unit rate, we divide the numerator by the denominator. This gives us a denominator of 1.

Example:
Exercise:

Problem:

Anita was paid $\$384$ last week for working 32 hours. What is Anita's hourly pay rate?

Solution:

Start with a rate of dollars to hours. Then divide.	$\$384$ last week for 32 hours
Write as a rate.	$\frac{\$384}{32\text{ hours}}$
Divide the numerator by the denominator.	$\frac{\$12}{1\text{ hour}}$
Rewrite as a rate.	$\$12/\text{hour}$

Anita's hourly pay rate is $12 per hour.

Note:
Try It
Exercise:

Problem: Find the unit rate: $630 for 35 hours.

Solution:

$18.00/hour

Note:
Try It
Exercise:

Problem: Find the unit rate: $684 for 36 hours.

Solution:

$19.00/hour

Example:
Exercise:

Problem:

Sven drives his car 455 miles, using 14 gallons of gasoline. How many miles per gallon does his car get?

Solution:

Start with a rate of miles to gallons. Then divide.

	455 miles to 14 gallons of gas
Write as a rate.	$\frac{455 \text{ miles}}{14 \text{ gallons}}$
Divide 455 by 14 to get the unit rate.	$\frac{32.5 \text{ miles}}{1 \text{ gallon}}$

Sven's car gets 32.5 miles/gallon, or 32.5 mpg.

Note:
Try It
Exercise:

Problem: Find the unit rate: 423 miles to 18 gallons of gas.

Solution:

23.5 mpg

Note:
Try It
Exercise:

Problem: Find the unit rate: 406 miles to 14.5 gallons of gas.

Solution:

28 mpg

Find Unit Price

Sometimes we buy common household items 'in bulk', where several items are packaged together and sold for one price. To compare the prices of different sized packages, we need to find the unit price. To find the unit price, divide the total price by the number of items. A **unit price** is a unit rate for one item.

Note:
Unit price
A **unit price** is a unit rate that gives the price of one item.

Example:
Exercise:

Problem:

The grocery store charges $\$3.99$ for a case of 24 bottles of water. What is the unit price?

Solution:

What are we asked to find? We are asked to find the unit price, which is the price per bottle.

Write as a rate.	$\frac{\$3.99}{24 \text{ bottles}}$
Divide to find the unit price.	$\frac{\$0.16625}{1 \text{ bottle}}$
Round the result to the nearest penny.	$\frac{\$0.17}{1 \text{ bottle}}$

The unit price is approximately $\$0.17$ per bottle. Each bottle costs about $\$0.17$.

Note:
Try It
Exercise:

Problem: Find the unit price. Round your answer to the nearest cent if necessary.

24-pack of juice boxes for $6.99

Solution:

$0.29/box

Note:
Try It
Exercise:

Problem: Find the unit price. Round your answer to the nearest cent if necessary.

24-pack of bottles of ice tea for $12.72

Solution:

$0.53/bottle

Unit prices are very useful if you comparison shop. The *better buy* is the item with the lower unit price. Most grocery stores list the unit price of each item on the shelves.

Example:
Exercise:

Problem:

Paul is shopping for laundry detergent. At the grocery store, the liquid detergent is priced at $14.99 for 64 loads of laundry and the same brand of powder detergent is priced at $15.99 for 80 loads.

Which is the better buy, the liquid or the powder detergent?

Solution:

To compare the prices, we first find the unit price for each type of detergent.

	Liquid	Powder

Write as a rate.	$\frac{\$14.99}{64\ \text{loads}}$	$\frac{\$15.99}{80\ \text{loads}}$
Find the unit price.	$\frac{\$0.234\ldots}{1\ \text{load}}$	$\frac{\$0.199\ldots}{1\ \text{load}}$
Round to the nearest cent.	$\$0.23/\text{load}$ (23 cents per load.)	$\$0.20/\text{load}$ (20 cents per load)

Now we compare the unit prices. The unit price of the liquid detergent is about $\$0.23$ per load and the unit price of the powder detergent is about $\$0.20$ per load. The powder is the better buy.

Note:
Try It
Exercise:

Problem:

Find each unit price and then determine the better buy. Round to the nearest cent if necessary.

Brand A Storage Bags, \$4.59 for 40 count, or Brand B Storage Bags, \$3.99 for 30 count

Solution:

Brand A costs \$0.12 per bag. Brand B costs \$0.13 per bag. Brand A is the better buy.

Note:
Try It
Exercise:

Problem:

Find each unit price and then determine the better buy. Round to the nearest cent if necessary.

Brand C Chicken Noodle Soup, \$1.89 for 26 ounces, or Brand D Chicken Noodle Soup, \$0.95 for 10.75 ounces

Solution:

Brand C costs \$0.07 per ounce. Brand D costs \$0.09 per ounce. Brand C is the better buy.

Notice in [link] that we rounded the unit price to the nearest cent. Sometimes we may need to carry the division to one more place to see the difference between the unit prices.

Translate Phrases to Expressions with Fractions

Have you noticed that the examples in this section used the comparison words *ratio of, to, per, in, for, on,* and *from*? When you translate phrases that include these words, you should think either ratio or rate. If the units measure the same quantity (length, time, etc.), you have a ratio. If the units are different, you have a rate. In both cases, you write a fraction.

Example:
Exercise:

Problem: Translate the word phrase into an algebraic expression:

a. 427 miles per h hours

b. x students to 3 teachers

c. y dollars for 18 hours

Solution:

a	
	427 miles per h hours
Write as a rate.	$\frac{427\text{ miles}}{h\text{ hours}}$

b	
	x students to 3 teachers
Write as a rate.	$\frac{x\text{ students}}{3\text{ teachers}}$

c	
	y dollars for 18 hours
Write as a rate.	$\frac{\$y}{18\text{ hours}}$

Note:
Try It
Exercise:

Problem: Translate the word phrase into an algebraic expression.

a. 689 miles per h hours
b. y parents to 22 students
c. d dollars for 9 minutes

Solution:

689 mi/h hours
y parents/22 students
$\$d$/9 min

Note:

Try It
Exercise:

Problem: Translate the word phrase into an algebraic expression.

a. m miles per 9 hours
b. x students to 8 buses
c. y dollars for 40 hours

Solution:

m mi/9 h
x students/8 buses
$\$y$/40 h

Homework

Write a Ratio as a Fraction

In the following exercises, write each ratio as a fraction and simplify.
Exercise:

Problem: 20 to 36

Solution:

$\frac{5}{9}$

Exercise:

Problem: 20 to 32

Exercise:

Problem: 42 to 48

Solution:

$\frac{7}{8}$

Exercise:

Problem: 45 to 54

Exercise:

Problem: 49 to 21

Solution:

$\frac{7}{3}$

Exercise:

Problem: 56 to 16

Exercise:

Problem: 84 to 36

Solution:

$\frac{7}{3}$

Exercise:

Problem: 6.4 to 0.8

Exercise:

Problem: 0.56 to 2.8

Solution:

$\frac{1}{5}$

Exercise:

Problem: 1.26 to 4.2

Exercise:

Problem: $1\frac{2}{3}$ to $2\frac{5}{6}$

Solution:

$\frac{10}{17}$

Exercise:

Problem: $1\frac{3}{4}$ to $2\frac{5}{8}$

Exercise:

Problem: $4\frac{1}{6}$ to $3\frac{1}{3}$

Solution:

$\frac{5}{4}$

Exercise:

Problem: $5\frac{3}{5}$ to $3\frac{3}{5}$

Exercise:

Problem: \$18 to \$63

Solution:

$\frac{2}{7}$

Exercise:

Problem: \$16 to \$72

Exercise:

Problem: \$1.21 to \$0.44

Solution:

$\frac{11}{4}$

Exercise:

Problem: \$1.38 to \$0.69

Exercise:

Problem: 28 ounces to 84 ounces

Solution:

$\frac{1}{3}$

Exercise:

Problem: 32 ounces to 128 ounces

Exercise:

Problem: 12 feet to 46 feet

Solution:

$\frac{6}{23}$

Exercise:

Problem: 15 feet to 57 feet

Exercise:

Problem: 246 milligrams to 45 milligrams

Solution:

$\frac{82}{15}$

Exercise:

Problem: 304 milligrams to 48 milligrams

Exercise:

Problem: total cholesterol of 175 to HDL cholesterol of 45

Solution:

$\frac{35}{9}$

Exercise:

Problem: total cholesterol of 215 to HDL cholesterol of 55

Exercise:

Problem:

Write the chemical-to-solution ratio when 480 mL of chemical is mixed with 720 mL of water. Then reduce to a denominator of one.

Solution:

$\frac{480}{1200} = \frac{0.4}{1}$

Exercise:

Problem:

Write the chemical-to-solution ratio when 2.5 L of chemical is mixed with 10 L of water. Then reduce to a denominator of one.

Exercise:

Problem: 27 inches to 1 foot

Solution:

$\frac{9}{4}$

Exercise:

Problem: 28 inches to 1 foot

Write a Rate as a Fraction

In the following exercises, write each rate as a fraction.

Exercise:

Problem: 140 calories per 12 ounces

Solution:

$\frac{35 \text{ calories}}{3 \text{ ounces}}$

Exercise:

Problem: 180 calories per 16 ounces

Exercise:

Problem: 8.2 pounds per 3 square inches

Solution:

$\frac{41 \text{ lbs}}{15 \text{ sq. in.}}$

Exercise:

Problem: 9.5 pounds per 4 square inches

Exercise:

Problem: 488 miles in 7 hours

Solution:

$\frac{488 \text{ miles}}{7 \text{ hours}}$

Exercise:

Problem: 527 miles in 9 hours

Exercise:

Problem: \$595 for 40 hours

Solution:

$\frac{\$119}{8 \text{ hours}}$

Exercise:

Problem: \$798 for 40 hours

Find Unit Rates

In the following exercises, find the unit rate. Round to two decimal places, if necessary.

Exercise:

Problem: 140 calories per 12 ounces

Solution:

11.67 calories/ounce

Exercise:

Problem: 180 calories per 16 ounces

Exercise:

Problem: 8.2 pounds per 3 square inches

Solution:

2.73 lbs./sq. in.

Exercise:

Problem: 9.5 pounds per 4 square inches

Exercise:

Problem: 488 miles in 7 hours

Solution:

69.71 mph

Exercise:

Problem: 527 miles in 9 hours

Exercise:

Problem: $595 for 40 hours

Solution:

$14.88/hour

Exercise:

Problem: $798 for 40 hours

Exercise:

Problem: 576 miles on 18 gallons of gas

Solution:

32 mpg

Exercise:

Problem: 435 miles on 15 gallons of gas

Exercise:

Problem: 43 pounds in 16 weeks

Solution:

2.69 lbs./week

Exercise:

Problem: 57 pounds in 24 weeks

Exercise:

Problem: 46 beats in 0.5 minute

Solution:

92 beats/minute

Exercise:

Problem: 54 beats in 0.5 minute

Exercise:

Problem:

The bindery at a printing plant assembles 96,000 magazines in 12 hours. How many magazines are assembled in one hour?

Solution:

8,000

Exercise:

Problem:

The pressroom at a printing plant prints 540,000 sections in 12 hours. How many sections are printed per hour?

Find Unit Price

In the following exercises, find the unit price. Round to the nearest cent.

Exercise:

Problem: Soap bars at 8 for $8.69

Solution:

$1.09/bar

Exercise:

Problem: Soap bars at 4 for $3.39

Exercise:

Problem: Women's sports socks at 6 pairs for $7.99

Solution:

$1.33/pair

Exercise:

Problem: Men's dress socks at 3 pairs for $8.49

Exercise:

Problem: Snack packs of cookies at 12 for $5.79

Solution:

$0.48/pack

Exercise:

Problem: Granola bars at 5 for $3.69

Exercise:

Problem: CD-RW discs at 25 for $14.99

Solution:

$0.60/disc

Exercise:

Problem: CDs at 50 for $4.49

Exercise:

Problem:

The grocery store has a special on macaroni and cheese. The price is $3.87 for 3 boxes. How much does each box cost?

Solution:

$1.29/box

Exercise:

Problem:

The pet store has a special on cat food. The price is $4.32 for 12 cans. How much does each can cost?

In the following exercises, find each unit price and then identify the better buy. Round to three decimal places.

Exercise:

Problem: Mouthwash, 50.7-ounce size for $6.99 or 33.8-ounce size for $4.79

Solution:

The 50.7-ounce size costs $0.138 per ounce. The 33.8-ounce size costs $0.142 per ounce. The 50.7-ounce size is the better buy.

Exercise:

Problem: Toothpaste, 6-ounce size for $3.19 or 7.8-ounce size for $5.19

Exercise:

Problem: Breakfast cereal, 18 ounces for \$3.99 or 14 ounces for \$3.29

Solution:

The 18-ounce size costs \$0.222 per ounce. The 14-ounce size costs \$0.235 per ounce. The 18-ounce size is a better buy.

Exercise:

Problem: Breakfast Cereal, 10.7 ounces for \$2.69 or 14.8 ounces for \$3.69

Exercise:

Problem:

Ketchup, 40-ounce regular bottle for \$2.99 or 64-ounce squeeze bottle for \$4.39

Solution:

The regular bottle costs \$0.075 per ounce. The squeeze bottle costs \$0.069 per ounce. The squeeze bottle is a better buy.

Exercise:

Problem:

Mayonnaise 15-ounce regular bottle for \$3.49 or 22-ounce squeeze bottle for \$4.99

Exercise:

Problem: Cheese \$6.49 for 1 lb. block or \$3.39 for $\frac{1}{2}$ lb. block

Solution:

The half-pound block costs \$6.78/lb, so the 1-lb. block is a better buy.

Exercise:

Problem: Candy \$10.99 for a 1 lb. bag or \$2.89 for $\frac{1}{4}$ lb. of loose candy

Translate Phrases to Expressions with Fractions

In the following exercises, translate the English phrase into an algebraic expression.
Exercise:

Problem: 793 miles per p hours

Solution:

$\frac{793 \text{ miles}}{p \text{ hours}}$

Exercise:

Problem: 78 feet per r seconds

Exercise:

Problem: $\$3$ for 0.5 lbs.

Solution:

$\frac{\$3}{0.5 \text{ lbs.}}$

Exercise:

Problem: j beats in 0.5 minutes

Exercise:

Problem: 105 calories in x ounces

Solution:

$\frac{105 \text{ calories}}{x \text{ ounces}}$

Exercise:

Problem: 400 minutes for m dollars

Exercise:

Problem: the ratio of y to $5x$

Solution:

$\frac{y}{5x}$

Exercise:

Problem: the ratio of $12x$ to y

Exercise:

Problem:

One elementary school in Ohio has 684 students and 45 teachers. Write the student-to-teacher ratio as a unit rate.

Solution:

15.2 students per teacher

Exercise:

Problem:

If the average American produces about $1,600$ pounds of paper trash per year (365 days). How many pounds of paper trash does the average American produce each day? (Round to the nearest tenth of a pound.)

Exercise:

Problem:

A popular fast food burger weighs 7.5 ounces and contains 540 calories, 29 grams of fat, 43 grams of carbohydrates, and 25 grams of protein. Find the unit rate of

a. calories per ounce
b. grams of fat per ounce
c. grams of carbohydrates per ounce
d. grams of protein per ounce.

Solution:

72 calories/ounce
3.87 grams of fat/ounce
5.73 grams carbs/ounce
3.33 grams protein/ounce

Exercise:

Problem:

A 16-ounce chocolate mocha coffee with whipped cream contains 470 calories, 18 grams of fat, 63 grams of carbohydrates, and 15 grams of protein. Find the unit rate of

a. calories per ounce
b. grams of fat per ounce
c. grams of carbohydrates per ounce
d. grams of protein per ounce

The Order of Operations
A review of the order of operations, using whole numbers, in preparation to working with integers and real numbers.

Grouping Symbols

Grouping symbols are used to indicate that a particular collection of numbers and meaningful operations are to be grouped together and considered as one number. The grouping symbols commonly used in mathematics are the following:

Parentheses: ( )

Brackets: [ ]

Braces: { }

Fraction Bar: —

In a computation in which more than one operation is involved, grouping symbols indicate which operation to perform first. If possible, we perform operations inside grouping symbols first.

For example:

$(5 \cdot 5) + 20 = 45$ since we must multiply 5 times 5 to obtain 25 first and then add the 20.

whereas:

$5 \cdot (5 + 20) = 125$ since we must add 5 and 20 first to obtain 25 and then multiply that result times 5.

Grouping Symbols Examples

If possible, determine the value of each of the following.

Example 1

9 + (3 · 8)

Since 3 and 8 are within parentheses, they are to be combined first:

= $9 + 24$

Then add the terms:

= 33

Thus, 9 + (3 · 8) = 33.

Example 2

(10 ÷ 0) · 6

Since (10 ÷ 0) is undefined, we attach no value to it. We write, “undefined.”

Grouping Symbols Practice Exercises

If possible, determine the value of each of the following.

Note:
Try It
Exercise:

Problem: 16 – (3 · 2)

Solution:

10

Note:
Try It
Exercise:

Problem: $5 + (7 \cdot 9)$

Solution:

68

Note:
Try It
Exercise:

Problem: $(4 + 8) \cdot 2$

Solution:

24

Note:
Try It
Exercise:

Problem: $28 \div (18 - 11)$

Solution:

4

Note:
Try It
Exercise:

Problem: $(33 \div 3) – 11$

Solution:

0

Note:
Try It
Exercise:

Problem: $4 + (0 \div 0)$

Solution:

undefined

Multiple Grouping Symbols

When a set of grouping symbols occurs inside another set of grouping symbols, we perform the operations within the innermost set first.

When simplifying expressions with multiple operations, you must perform all multiplications and divisions first (in order as you move from left to right) followed by all additions and subtractions (in order as you move from left to right).

Multiple Grouping Symbol Examples

Determine the value of each of the following.

Example:
$2 + (8 \cdot 3) - (5 + 6)$
Combine 8 and 3 first, then combine 5 and 6.
$= 2 + 24 - 11$
Now combine left to right.
$= 26 - 11$
$= 15$

Example:
$10 + [30 - (2 \cdot 9)]$
Combine 2 and 9 since they occur in the innermost set of parentheses.
$= 10 + [30 - 18]$
Now combine 30 and 18.
$= 10 + 12$
$= 22$

Multiple Grouping Symbol Practice Exercises

Determine the value of each of the following:

Note:

Try It
Exercise:

Problem: $(17 + 8) + (9 + 20)$

Solution:

54

Note:
Try It
Exercise:

Problem: $(55 - 6) + (13 \cdot 2)$

Solution:

75

Note:
Try It
Exercise:

Problem: $23 + (12 \div 4) + (11 \cdot 2)$

Solution:

48

Note:
Try It
Exercise:

Problem: $86 + [14 + (10 - 8)]$

Solution:

102

Note:
Try It
Exercise:

Problem: $31 + (9 + [1 + (35 - 2)])$

Solution:

74

Order of Operations (PEMDAS)

Sometimes there are no grouping symbols indicating which operations to perform first. For example, suppose we wish to find the value of $3 + 5 \cdot 2$. We could do either of two things:

Add 3 and 5, then multiply this sum by 2.

$$\begin{aligned} &3 + 5 \cdot 2 \\ &= 8 \cdot 2 \\ &= 16 \end{aligned}$$

Multiply 5 and 2, then add 3 to this product.

$$\begin{aligned} & 3 + 5 \cdot 2 \\ &= 3 + 10 \\ &= 13 \end{aligned}$$

We now have two values for the same expression.

We need a set of rules to guide everyone to one unique value for this kind of expression.

The universally agreed-upon **order of operations**, often referred to as PEMDAS, for evaluating a mathematical expression is as follows:

P: Parentheses (grouping symbols) from the inside out.

By parentheses we mean anything that acts as a grouping symbol, including anything inside symbols such as [ ], { }, | |, and $\sqrt{\ }$.

Any expression in the numerator or denominator of a fraction or in an exponent is also considered grouped, and should be simplified before carrying out further operations.

If there are nested parentheses (parentheses inside parentheses), you work from the innermost parentheses outward.

E: Exponents and other special functions, such as $\sqrt{\ }$ or trigonometric or logarithmic functions .

MD: Multiplications and Divisions, in order as you move from left to right.

AS: Additions and Subtractions, in order as you move from left to right.

For example, given: $3 + 15 \div 3 + 5 \times 2^{2+3}$

The exponent is an implied grouping, so the 2+3 must be evaluated first:

$= 3 + 15 \div 3 + 5 \times 2^5$

Now the exponent is carried out:

$= 3 + 15 \div 3 + 5 \times 32$

Then the multiplication and division, left to right using $15 \div 3 = 5$ and $5 \times 32 = 160$:

$= 3 + 5 + 160$

Finally, the addition, left to right:

$= 168$

Examples, Order of Operations

Determine the value of each of the following.

Example:
$21 + 3 \cdot 12.$
Multiply first:
$= 21 + 36$
Add.
$= 57$

Example:
$(15 - 8) + 5(6 + 4).$
Simplify inside parentheses first.

$= 7 + 5 \cdot 10$
Multiply.
$= 7 + 50$
Add.
= 57

Example:
$63 - (4 + 6 \cdot 3) + 76 - 4.$
Simplify first within the parentheses by multiplying, then adding:
$= 63 - (4 + 18) + 76 - 4$
$= 63 - 22 + 76 - 4$
Now perform the additions and subtractions, moving left to right:
$= 41 + 76 - 4$
$= 117 - 4$
= 113.

Example:
$7 \cdot 6 - 4^2 + 1^5$
Evaluate the exponents.
$= 7 \cdot 6 - 16 + 1$
Multiply $7 \cdot 6$:
$= 42 - 16 + 1$
Perform additions and subtractions from left to right. Subtraction is first, so subtract 16 from 42:
= 26 + 1
Add 26 and 1:

= 27.

Example:
$6 \cdot (3^2 + 2^2) + 4^2$
Evaluate the exponents in the parentheses:
= $6 \cdot (9 + 4) + 4^2$
Add 9 and 4 in the parentheses:
= $6 \cdot (13) + 4^2$
Evaluate the exponential form 4^2:
= $6 \cdot (13) + 16$
Multiply 6 and 13:
= $78 + 16$
Add 78 and 16:
= 94

Example:
$\frac{6^2+2^2}{4^2+6 \cdot 2^2} + \frac{1^3+8^2}{10^2-19 \cdot 5}$.
Recall that the fraction bar is a grouping symbol. The fraction $\frac{6^2+2^2}{4^2+6 \cdot 2^2}$ is equivalent to $\left(6^2 + 2^2\right) \div \left(4^2 + 6 \cdot 2^2\right)$
Evaluate all exponents.
= $\frac{36+4}{16+6 \cdot 4} + \frac{1+64}{100-19 \cdot 5}$
Perform all multiplications.
= $\frac{36+4}{16+24} + \frac{1+64}{100-95}$

Perform all additions and subtractions that are above or below the fraction bars.
$= \frac{40}{40} + \frac{65}{5}$
Perform all divisions, denoted by the fraction bars.
= 1+13
Add.
= 14

Order of Operations, Practice Exercises

Determine the value of the following:

Note:
Try It
Exercise:

Problem: 2{8 + (32 – 7)}

Solution:

66

Note:
Try It
Exercise:

Problem: (34 + 18 – 2 · 3) + 11

Solution:

57

Note:
Try It
Exercise:

Problem: $8(10) + 4(2 + 3) - (20 + 3 \cdot 15 + 40 - 5)$

Solution:

0

Note:
Try It
Exercise:

Problem: $5 \cdot 8 + 42 - 22$

Solution:

60

Note:
Try It
Exercise:

Problem: $4(6^2 - 3^3) \div (4^2 - 4)$

Solution:

3

Note:
Try It
Exercise:

Problem: $\{6 – [24 \div (4 \cdot 2)]\}^3$

Solution:

27

Note:
Try It
Exercise:

Problem: $(8 + 9 \cdot 3) \div 7 + 5 \cdot (8 \div 4 + 7 + 3 \cdot 5)$

Solution:

125

Homework

For the following problems, find each value.
Exercise:

Problem: $1 - 5(8 - 8)$

Solution:

1

Exercise:

Problem: $37 - 1 \cdot 6^2$

Exercise:

Problem: $98 \div 2 \div 7^2$

Solution:

1

Exercise:

Problem: $(4^2 - 2 \cdot 4) - 2^3$

Exercise:

Problem: $61 - 22 + 4[3 \cdot 10 + 11]$

Solution:

203

Exercise:

Problem: $121 - 4 \cdot [4 \cdot 5 - 12] + \frac{16}{2}$

Exercise:

Problem: $2^2 \cdot 3 + 2^3(6 - 2) - \left(3 + 17\right) + 11(6)$

Solution:

90

Exercise:

Problem: $\frac{8(6+20)}{4} + \frac{3(6+16)}{11}$

Exercise:

Problem: $\frac{1+16-3}{7} + 5(12)$

Solution:

62

Exercise:

Problem: $1^6 + 0^8 + 5^2(2+8)^3$

Exercise:

Problem: $\frac{5(8^2-9\cdot 6)}{2^5-7} + \frac{7^2-4^2}{2^4-5}$

Solution:

5

Exercise:

Problem: $6\{2\cdot 8+3\}-5\cdot 2+\frac{8}{4}+(1+8)\cdot(1+11)$

Exercise:

Problem: $26-2\cdot\left\{\frac{6+20}{13}\right\}$

Solution:

22

Exercise:

Problem: $(10+5)\cdot(10+5)-4\cdot(60-4)$

Exercise:

Problem: $\frac{6^2-1}{2^3-3}+\frac{4^3+2\cdot3}{2\cdot5}$

Solution:

14

Exercise:

Problem: $\frac{(2+1)^3+2^3+1^{10}}{6^2}-\frac{15^2-[2\cdot5]^2}{5\cdot5^2}$

Adding Integers

Learning Objectives

By the end of this section, you will be able to:

- Model addition of integers
- Simplify expressions with integers

Model Addition of Integers

Most students are comfortable with the addition and subtraction facts for positive numbers. But doing addition or subtraction with both positive and negative numbers may be more difficult. This difficulty relates to the way the brain learns.

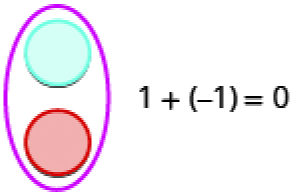
The brain learns best by working with objects in the real world and then generalizing to abstract concepts. Toddlers learn quickly that if they have two cookies and their older brother steals one, they have only one left. This is a concrete example of $2-1$. Children learn their basic addition and subtraction facts from experiences in their everyday lives. Eventually, they know the number facts without relying on cookies.

Addition and subtraction of negative numbers have fewer real world examples that are meaningful to us. Math teachers have several different approaches, such as number lines, banking, temperatures, and so on, to make these concepts real.

We will model addition and subtraction of negatives with two color counters. We let a blue counter represent a positive and a red counter will represent a negative.

If we have one positive and one negative counter, the value of the pair is zero. They form a neutral pair. The value of this neutral pair is zero as

summarized in [link].



A blue counter represents $+1$. A red counter represents -1. Together they add to zero.

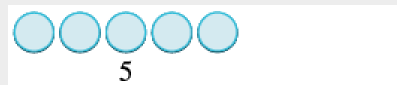
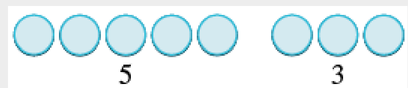
We will model four addition facts using the numbers $5, -5$ and 3 , -3.
Equation:

$$5+3 \quad -5+(-3) \quad -5+3 \quad 5+(-3)$$

Example:
Exercise:

Problem: Model: $5+3$.

Solution:

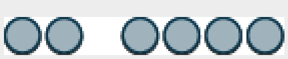
Interpret the expression.	$5+3$ means the sum of 5 and 3.
Model the first number. Start with 5 positives.	
Model the second number. Add 3 positives.	
Count the total number of counters.	
The sum of 5 and 3 is 8.	$5+3=8$

Note:
Try It
Exercise:

Problem: Model the expression.

$2+4$

Solution:



2 + 4 = 6

Note:
Try It
Exercise:

Problem: Model the expression.

$2 + 5$

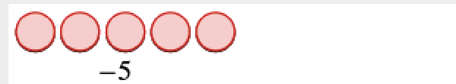
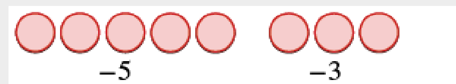
Solution:

2 + 5 = 7

Example:
Exercise:

Problem: Model: $-5 + (-3)$.

Solution:

Interpret the expression.	$-5+(-3)$ means the sum of -5 and -3.
Model the first number. Start with 5 negatives.	
Model the second number. Add 3 negatives.	
Count the total number of counters.	
The sum of −5 and −3 is −8.	$-5+-3=-8$

Note:
Try It
Exercise:

Problem: Model the expression.

$-2+(-4)$

Solution:

-2 + (-4) = −6

Note:
Try It
Exercise:

Problem: Model the expression.

$-2+(-5)$

Solution:

-2 + (-5) = −7

[link] and [link] are very similar. The first example adds 5 positives and 3 positives—both positives. The second example adds 5 negatives and 3 negatives—both negatives. In each case, we got a result of 8—either 8 positives or 8 negatives. When the signs are the same, the counters are all the same color.

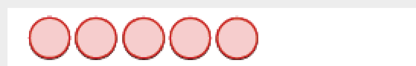
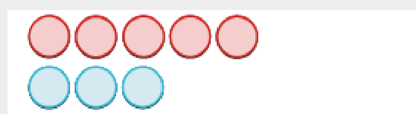
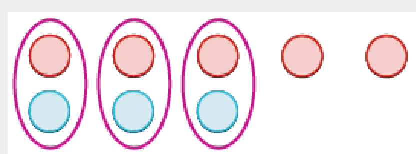
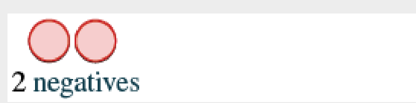
Now let’s see what happens when the signs are different.

Example:

Exercise:

Problem: Model: $-5+3$.

Solution:

Interpret the expression.	$-5+3$ means the sum of -5 and 3.
Model the first number. Start with 5 negatives.	
Model the second number. Add 3 positives.	
Remove any neutral pairs.	
Count the result.	
The sum of −5 and 3 is −2.	$-5+3=-2$

Notice that there were more negatives than positives, so the result is negative.

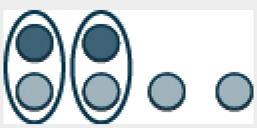
Note:
Try It
Exercise:

Problem: Model the expression, and then simplify:

$2 + (-4)$

Solution:

Note: Dark blue circles represent negatives and light blue counters represent positives.



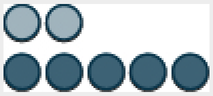
2 + (-4) = -2

Note:
Try It
Exercise:

Problem: Model the expression, and then simplify:

$2 + (-5)$

Solution:

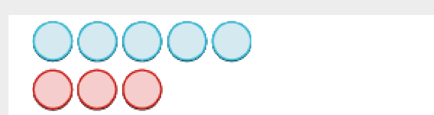


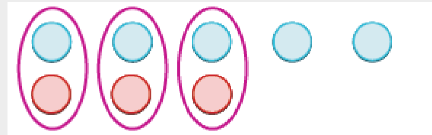
2 + (-5) = -3

Example:
Exercise:

Problem: Model: $5+(-3)$.

Solution:

Interpret the expression.	$5+(-3)$ means the sum of 5 and -3.
Model the first number. Start with 5 positives.	
Model the second number. Add 3 negatives.	

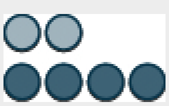
Remove any neutral pairs.	
Count the result.	
The sum of 5 and −3 is 2.	$5 + (-3) = 2$

Note:
Try It
Exercise:

Problem: Model the expression, and then simplify:

$2 + (-4)$

Solution:



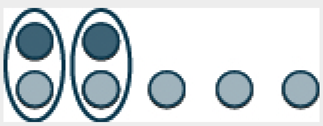
2 + (-4) = -2

Note:
Try It
Exercise:

Problem: Model the expression:

$-2+5$

Solution:



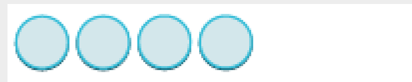
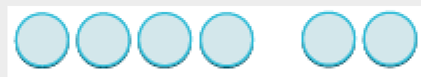
-2 + 5 = 3

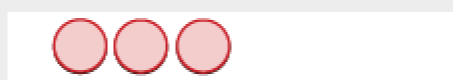
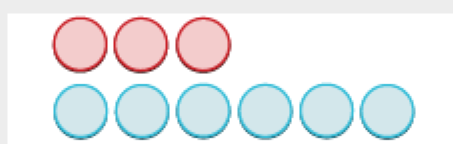
Example:
Modeling Addition of Positive and Negative Integers
Model each addition.
Exercise:

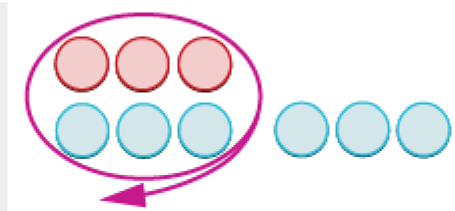
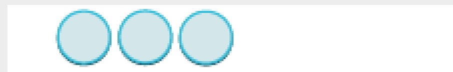
Problem:

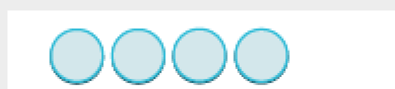
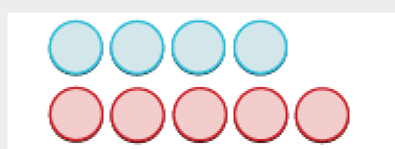
4 + 2
−3 + 6
4 + (−5)
-2 + (−3)

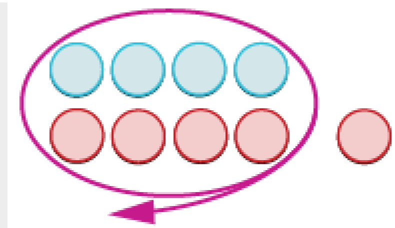
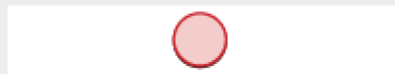
Solution:

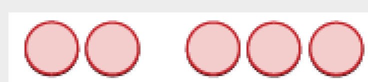
a	
	$4+2$
Start with 4 positives.	
Add two positives.	
How many do you have?	$4+2=6$

b	
	$-3+6$
Start with 3 negatives.	
Add 6 positives.	
Remove neutral pairs.	

	
How many are left?	
	$-3 + 6 = 3$

c	
	$4 + (-5)$
Start with 4 positives.	
Add 5 negatives.	
Remove neutral pairs.	

	
How many are left?	
	$4 + (-5) = -1$

d	
	$-2 + (-3)$
Start with 2 negatives.	
Add 3 negatives.	
How many do you have?	$-2 + (-3) = -5$

Note:
Try It
Exercise:

Problem: Model each addition.

3 + 4
−1 + 4
4 + (−6)
−2 + (−2)

Solution:

3 + 4 = 7

-1 + 4 = 3

4 + (-6) = -2

-2 + (-2) = -4

Note:
Try It
Exercise:

Problem:

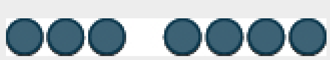
5 + 1
−3 + 7
2 + (−8)
−3 + (−4)

Solution:

5 + 1 = 6

-3 + 7 = 4

2 + (-8) = -6



-3 + (-4) = -7

Simplify Expressions with Integers

Now that you have modeled adding small positive and negative integers, you can visualize the model in your mind to simplify expressions with any integers.

For example, if you want to add $37+(-53)$, you don't have to count out 37 blue counters and 53 red counters.

Picture 37 blue counters with 53 red counters lined up underneath. Since there would be more negative counters than positive counters, the sum would be negative. Because $53-37=16$, there are 16 more negative counters.
Equation:

$$37+(-53)=-16$$

Let's try another one. We'll add $-74+(-27)$. Imagine 74 red counters and 27 more red counters, so we have 101 red counters all together. This means the sum is -101.
Equation:

$$-74+(-27)=-101$$

$5+3$	$-5+(-3)$
both positive, sum positive	both negative, sum negative

When the signs are the same, the counters would be all the same color, so add them.	
$-5+3$	$5+(-3)$
different signs, more negatives	different signs, more positives
Sum negative	sum positive
When the signs are different, some counters would make neutral pairs; subtract to see how many are left.	

Summary of the Addition of Positive and Negative Integers

Example:
Exercise:

Problem: Simplify:

$19+(-47)$
$-32+40$

Solution:

a. Since the signs are different, we subtract 19 from 47. The answer will be negative because there are more negatives than positives.
Equation:

$$19+(-47)$$
$$-28$$

b. The signs are different so we subtract 32 from 40. The answer will be positive because there are more positives than negatives.

Equation:

$$-32+40$$
$$8$$

Note:
Try It
Exercise:

Problem: Simplify each expression:

$15+(-32)$
$-19+76$

Solution:

−17
57

Note:
Try It
Exercise:

Problem: Simplify each expression:

$-55+9$
$43+(-17)$

Solution:

−46
26

Example:
Exercise:

Problem: Simplify: $-14 + (-36)$.

Solution:

Since the signs are the same, we add. The answer will be negative because there are only negatives.
Equation:

$$-14 + (-36)$$
$$-50$$

Note:
Try It
Exercise:

Problem: Simplify the expression:

$-31 + (-19)$

Solution:

−50

Note:
Try It
Exercise:

Problem: Simplify the expression:

$-42 + (-28)$

Solution:

-70

The techniques we have used up to now extend to more complicated expressions. Remember to follow the order of operations.

Example:
Exercise:

Problem: Simplify: $-5 + 3(-2 + 7)$.

Solution:

	$-5 + 3(-2 + 7)$
Simplify inside the parentheses.	$-5 + 3(5)$
Multiply.	$-5 + 15$

Add.	10

Note:
Try It
Exercise:

Problem: Simplify the expression:

$-2 + 5(-4 + 7)$

Solution:

13

Note:
Try It
Exercise:

Problem: Simplify the expression:

$-4 + 2(-3 + 5)$

Solution:

0

Summary

- **Addition of Positive and Negative Integers**

<table>
<tr><td>$5+3$</td><td>$-5+(-3)$</td></tr>
<tr><td>both positive, sum positive</td><td>both negative, sum negative</td></tr>
<tr><td colspan="2">When the signs are the same, the counters would be all the same color, so add them.</td></tr>
<tr><td>$-5+3$</td><td>$5+(-3)$</td></tr>
<tr><td>different signs, more negatives</td><td>different signs, more positives</td></tr>
<tr><td>Sum negative</td><td>sum positive</td></tr>
<tr><td colspan="2">When the signs are different, some counters would make neutral pairs; subtract to see how many are left.</td></tr>
</table>

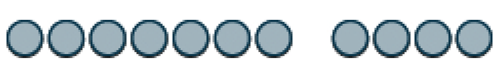
Homework

Model Addition of Integers

In the following exercises, model the expression to simplify.
Exercise:

Problem: $7+4$

Solution:



7 + 4 = 11

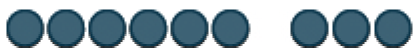
Exercise:

Problem: $8 + 5$

Exercise:

Problem: $-6 + (-3)$

Solution:



-6 + (-3) = -9

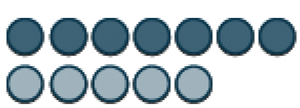
Exercise:

Problem: $-5 + (-5)$

Exercise:

Problem: $-7 + 5$

Solution:



-7 + 5 = -2

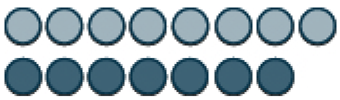
Exercise:

Problem: $-9+6$

Exercise:

Problem: $8+(-7)$

Solution:



8 + (-7) = 1

Exercise:

Problem: $9+(-4)$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $-21+(-59)$

Solution:

-80

Exercise:

Problem: $-35+(-47)$

Exercise:

Problem: $48 + (-16)$

Solution:

32

Exercise:

Problem: $34 + (-19)$

Exercise:

Problem: $-200 + 65$

Solution:

-135

Exercise:

Problem: $-150 + 45$

Exercise:

Problem: $2 + (-8) + 6$

Solution:

0

Exercise:

Problem: $4 + (-9) + 7$

Exercise:

Problem: $-14 + (-12) + 4$

Solution:

-22

Exercise:

Problem: $-17 + (-18) + 6$

Exercise:

Problem: $135 + (-110) + 83$

Solution:

108

Exercise:

Problem: $140 + (-75) + 67$

Exercise:

Problem: $-32 + 24 + (-6) + 10$

Solution:

-4

Exercise:

Problem: $-38 + 27 + (-8) + 12$

Exercise:

Problem: $19 + 2(-3 + 8)$

Solution:

29

Exercise:

Problem: $24 + 3(-5 + 9)$

Subtracting Integers

Learning Objectives

By the end of this section, you will be able to:

- Model subtraction of integers
- Simplify expressions with integers

Model Subtraction of Integers

Remember the story in the last section about the toddler and the cookies? Children learn how to subtract numbers through their everyday experiences. Real-life experiences serve as models for subtracting positive numbers, and in some cases, such as temperature, for adding negative as well as positive numbers. But it is difficult to relate subtracting negative numbers to common life experiences. Most people do not have an intuitive understanding of subtraction when negative numbers are involved. Math teachers use several different models to explain subtracting negative numbers.

We will continue to use counters to model subtraction. Remember, the blue counters represent positive numbers and the red counters represent negative numbers.

Perhaps when you were younger, you read $5-3$ as *five take away three*. When we use counters, we can think of subtraction the same way.

We will model four subtraction facts using the numbers 5 and 3.
Equation:

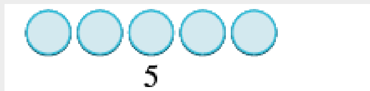
$$5-3 \quad -5-(-3) \quad -5-3 \quad 5-(-3)$$

Example:

Exercise:

Problem: Model: $5-3$.

Solution:

Interpret the expression.	$5-3$ means 5 take away 3.
Model the first number. Start with 5 positives.	
Take away the second number. So take away 3 positives.	
Find the counters that are left.	
	$5-3=2$. The difference between 5 and 3 is 2.

Note:

Try It
Exercise:

Problem: Model the expression:

$6 - 4$

Solution:

6 - 4 = 2

Note:
Try It
Exercise:

Problem: Model the expression:

$7 - 4$

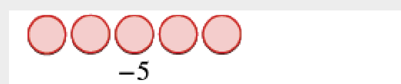
Solution:

7 - 4 = 3

Example:
Exercise:

Problem: Model: $-5-(-3)$.

Solution:

Interpret the expression.	$-5-(-3)$ means -5 take away -3.
Model the first number. Start with 5 negatives.	
Take away the second number. So take away 3 negatives.	
Find the number of counters that are left.	
	$-5-(-3)=-2$. The difference between

	-5 and -3 is -2.

Note:
Try It
Exercise:

Problem: Model the expression:

$-6 - (-4)$

Solution:

-6 - (-4) = −2

Note:
Try It
Exercise:

Problem: Model the expression:

$-7 - (-4)$

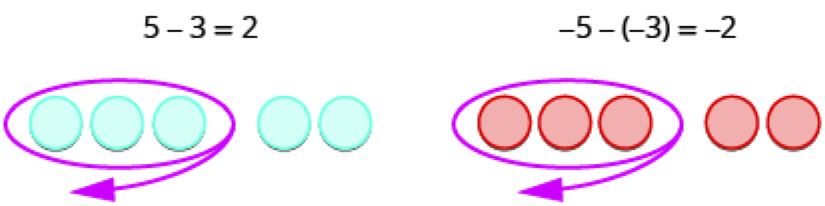
Solution:

-7 - (-4) = −3

Notice that [link] and [link] are very much alike.

- First, we subtracted 3 positives from 5 positives to get 2 positives.
- Then we subtracted 3 negatives from 5 negatives to get 2 negatives.

Each example used counters of only one color, and the “take away” model of subtraction was easy to apply.

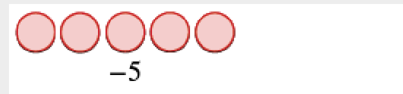
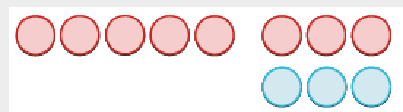
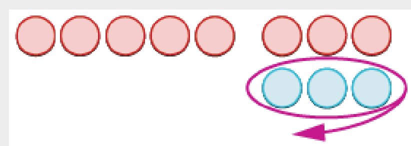


Now let’s see what happens when we subtract one positive and one negative number. We will need to use both positive and negative counters and sometimes some neutral pairs, too. Adding a neutral pair does not change the value.

Example:
Exercise:

Problem: Model: $-5-3$.

Solution:

Interpret the expression.	$-5 - 3$ means -5 take away 3.
Model the first number. Start with 5 negatives.	
Take away the second number. So we need to take away 3 positives.	
But there are no positives to take away. Add neutral pairs until you have 3 positives.	
Now take away 3 positives.	
Count the number of counters that are left.	
	$-5 - 3 = -8$. The difference of -5 and 3 is -8.

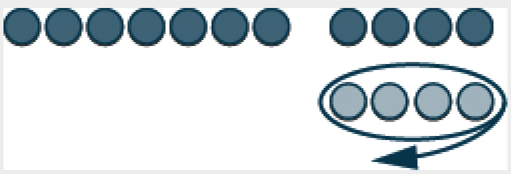
Note:
Try It

Exercise:

Problem: Model the expression:

$-7 - 4$

Solution:



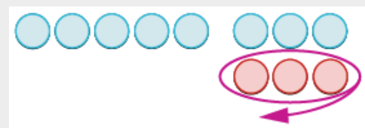
-7 - 4 = -11

Example:
Exercise:

Problem: Model: $5 - (-3)$.

Solution:

Interpret the expression.	$5 - (-3)$ means 5 take away -3.
Model the first number. Start with 5	

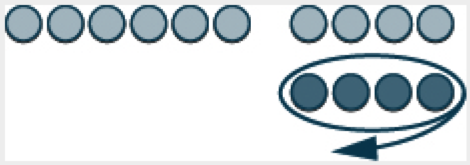
positives.	
Take away the second number, so take away 3 negatives.	
But there are no negatives to take away. Add neutral pairs until you have 3 negatives.	
Then take away 3 negatives.	
Count the number of counters that are left.	
The difference of 5 and -3 is 8.	$5-(-3)=8$

Note:
Try It
Exercise:

Problem: Model the expression:

$6-(-4)$

Solution:



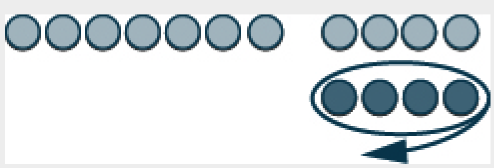
6 - (-4) = 10

Note:
Try It
Exercise:

Problem: Model the expression:

$7 - (-4)$

Solution:



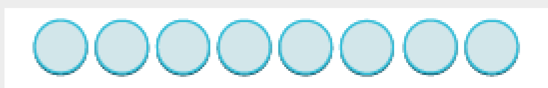
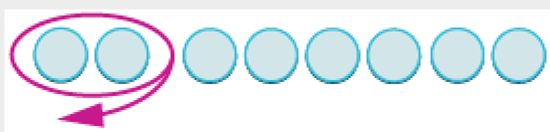
7 - (-4) = 11

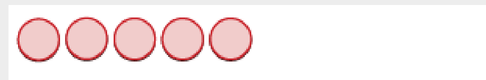
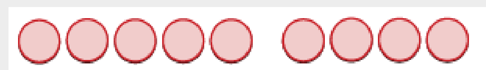
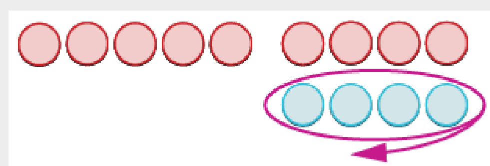
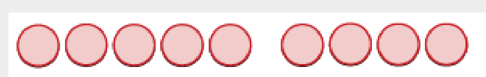
Example:
Exercise:

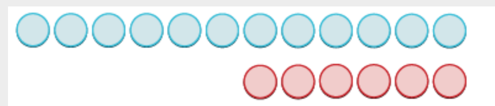
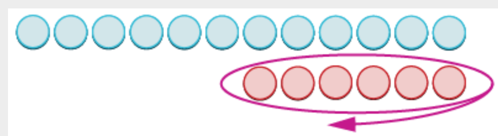
Problem: Model each subtraction.

8 − 2
−5 − 4
6 − (−6)
−8 − (−3)

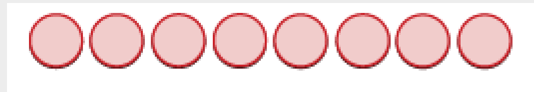
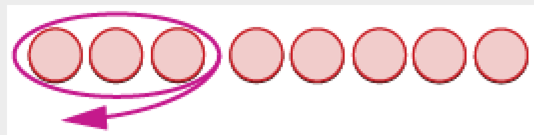
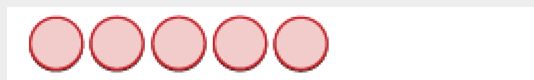
Solution:

a	
	$8 - 2$ means 8 take away 2.
Start with 8 positives.	
Take away 2 positives.	
How many are left?	6
	$8 - 2 = 6$

b	
	$-5 - 4$ means -5 take away 4.
Start with 5 negatives.	
You need to take away 4 positives.	
Add 4 neutral pairs to get 4 positives.	
Take away 4 positives.	
How many are left?	
	-9
	$-5 - 4 = -9$

c	
	$6 - (-6)$ means 6 take away -6.
Start with 6 positives.	
Add 6 neutrals to get 6 negatives to take away.	
Remove 6 negatives.	
How many are left?	
	12
	$6 - (-6) = 12$

d	
	$-8 - (-3)$ means -8 take away -3.

Start with 8 negatives.	
Take away 3 negatives.	
How many are left?	
	-5
	$-8-(-3)=-5$

Note:
Try It
Exercise:

Problem: Model each subtraction.

7 - (-8)
-2 - (-2)
4 - 1
-6 - 8

Solution:

a. 7 - (-8) = 15
b. -2 - (-2) = 0
c. 4 - 1 = 3
d. -6 - 8 = -14

Note:
Try It
Exercise:

Problem: Model each subtraction.

4 - (-6)
-8 - (-1)
7 - 3
-4 - 2

Solution:

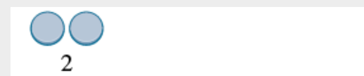
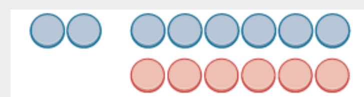
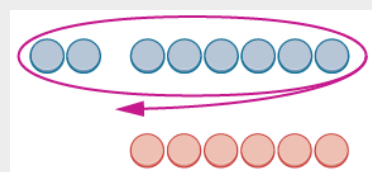
a. 4 - (-6) = 10
b. -8 - (-1) = -7
c. 7 - 3 = 4
d. -4 - 2 = -6

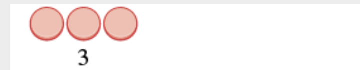
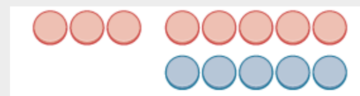
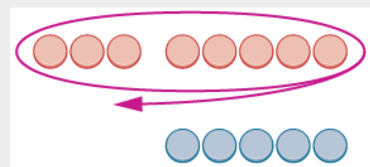
Example:
Exercise:

Problem: Model each subtraction expression:

$2 - 8$
$-3 - (-8)$

Solution:

a We start with 2 positives.	
We need to take away 8 positives, but we have only 2.	
Add neutral pairs until there are 8 positives to take away.	
Then take away eight positives.	
Find the number of counters that are left. There are 6 negatives.	
	$2 - 8 = -6$

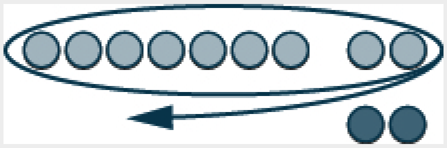
b We start with 3 negatives.	
We need to take away 8 negatives, but we have only 3.	
Add neutral pairs until there are 8 negatives to take away.	
Then take away the 8 negatives.	
Find the number of counters that are left. There are 5 positives.	
	$-3-(-8)=5$

Note:
Try It
Exercise:

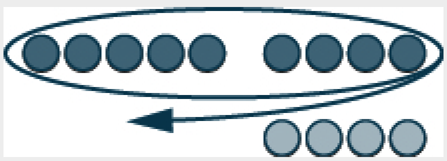
Problem: Model each subtraction expression.

$7-9$
$-5-(-9)$

Solution:

a. 

$7 - 9 = -2$

b. 

$-5 - (-9) = 4$

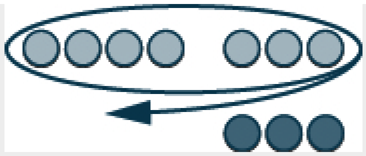
Note:
Try It
Exercise:

Problem: Model each subtraction expression.

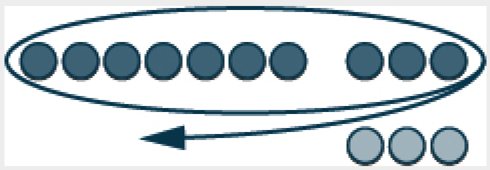
$4 - 7$
$-7 - (-10)$

Solution:

a.



4 - 7 = −3

b. 

-7 - (-10) = 3

Simplify Expressions with Integers

Do you see a pattern? Are you ready to subtract integers without counters? Let’s do two more subtractions. We’ll think about how we would model these with counters, but we won’t actually use the counters.

- Subtract $-23 - 7$.
  Think: We start with 23 negative counters.
  We have to subtract 7 positives, but there are no positives to take away.
  So we add 7 neutral pairs to get the 7 positives. Now we take away the 7 positives.
  So what’s left? We have the original 23 negatives plus 7 more negatives from the neutral pair. The result is 30 negatives.
  Equation:

$$-23 - 7 = -30$$

Notice, that to subtract 7, we added 7 negatives.

- Subtract $30-(-12)$.
  Think: We start with 30 positives.
  We have to subtract 12 negatives, but there are no negatives to take away.
  So we add 12 neutral pairs to the 30 positives. Now we take away the 12 negatives.
  What's left? We have the original 30 positives plus 12 more positives from the neutral pairs. The result is 42 positives.
  Equation:

$$30-(-12)=42$$

  Notice that to subtract -12, we added 12.

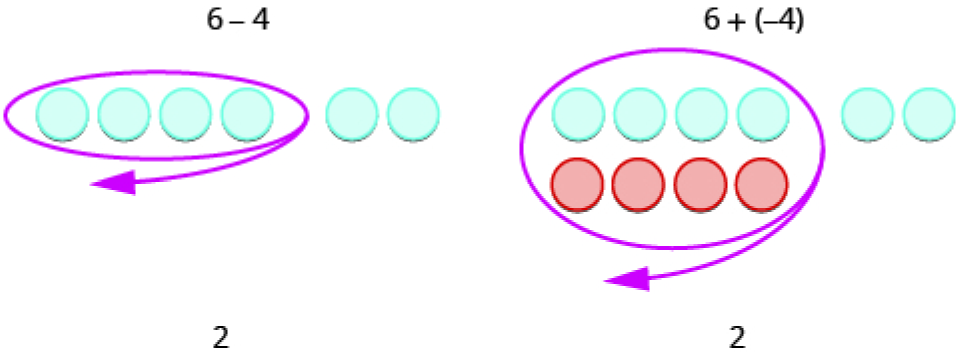
While we may not always use the counters, especially when we work with large numbers, practicing with them first gave us a concrete way to apply the concept, so that we can visualize and remember how to do the subtraction without the counters.

Have you noticed that subtraction of signed numbers can be done by adding the opposite? You will often see the idea, the Subtraction Property, written as follows:

Note:
Subtraction Property
Equation:

$$a-b=a+(-b)$$

Look at these two examples.



We see that $6-4$ gives the same answer as $6+(-4)$.

Of course, when we have a subtraction problem that has only positive numbers, like the first example, we just do the subtraction. We already knew how to subtract $6-4$ long ago. But knowing that $6-4$ gives the same answer as $6+(-4)$ helps when we are subtracting negative numbers.

Example:
Exercise:

Problem: Simplify:

$13-8$ and $13+(-8)$
$17-9$ and $-17+(-9)$

Solution:

a	
	$13-8$ and $13+(-8)$

Subtract to simplify.	$13 - 8 = 5$
Add to simplify.	$13 + (-8) = 5$
Subtracting 8 from 13 is the same as adding −8 to 13.	

b	
	$-17 - 9$ and $-17 + (-9)$
Subtract to simplify.	$-17 - 9 = -26$
Add to simplify.	$-17 + (-9) = -26$
Subtracting 9 from −17 is the same as adding −9 to −17.	

Note:
Try It
Exercise:

Problem: Simplify each expression:

$21 - 13$ and $21 + (-13)$
$-11 - 7$ and $-11 + (-7)$

Solution:

8, 8
−18, −18

Note:
Try It
Exercise:

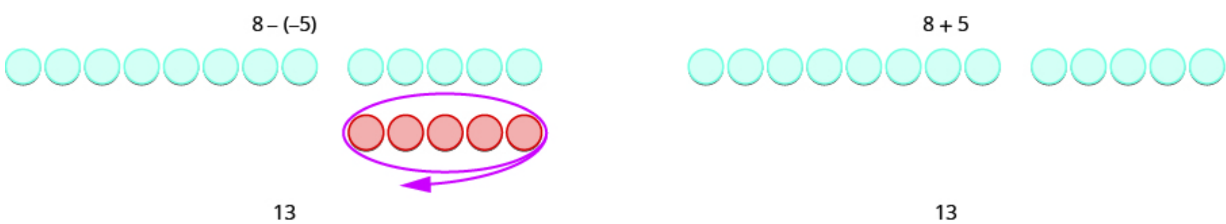
Problem: Simplify each expression:

$15-7$ and $15+(-7)$
$-14-8$ and $-14+(-8)$

Solution:

8, 8
−22, −22

Now look what happens when we subtract a negative.



We see that $8-(-5)$ gives the same result as $8+5$. Subtracting a negative number is like adding a positive.

Example:
Exercise:

Problem: Simplify:

$9-(-15)$ and $9+15$
$-7-(-4)$ and $-7+4$

Solution:

a	
	$9-(-15)$ and $9+15$
Subtract to simplify.	$9-(-15)=-24$
Add to simplify.	$9+15=24$
9−(−15) Subtracting −15 from 9 is the same as adding 15 to 9.	

b	
	$-7-(-4)$ and $-7+4$

Subtract to simplify.	$-7-(-4)=-3$
Add to simplify.	$-7+4=-3$
Subtracting −4 from −7 is the same as adding 4 to −7	

Note:
Try It
Exercise:

Problem: Simplify each expression:

$6-(-13)$ and $6+13$
$-5-(-1)$ and $-5+1$

Solution:

19, 19
−4, −4

Note:
Try It
Exercise:

Problem: Simplify each expression:

$4-(-19)$ and $4+19$

$-4-(-7)$ and $-4+7$

Solution:

23, 23
3, 3

$5-3$	$-5-(-3)$
2	-2
2 positives	2 negatives
When there would be enough counters of the color to take away, subtract.	
$-5-3$	$5-(-3)$
-8	8
5 negatives, want to subtract 3 positives	5 positives, want to subtract 3 negatives
need neutral pairs	need neutral pairs
When there would not be enough of the counters to take away, add neutral pairs.	

Summary of Subtraction of Integers

Example:
Exercise:

Problem: Simplify: $-74-(-58)$.

Solution:

We are taking 58 negatives away from 74 negatives.	$-74-(-58)$
Subtract.	-16

Note:
Try It
Exercise:

Problem: Simplify the expression:

$-67-(-38)$

Solution:

-29

Note:
Try It

Exercise:

Problem: Simplify the expression:

$-83 - (-57)$

Solution:

-26

Example:
Exercise:

Problem: Simplify: $7 - (-4 - 3) - 9$.

Solution:

We use the order of operations to simplify this expression, performing operations inside the parentheses first. Then we subtract from left to right.

Simplify inside the parentheses first.	$7 - (-4 - 3) - 9$
Subtract from left to right.	$7 - (-7) - 9$
Subtract.	

	$14 - 9$
	5

Note:
Try It
Exercise:

Problem: Simplify the expression:

$8 - (-3 - 1) - 9$

Solution:

3

Note:
Try It
Exercise:

Problem: Simplify the expression:

$12 - (-9 - 6) - 14$

Solution:

13

Example:
Exercise:

Problem: Simplify: $3 \cdot 7 - 4 \cdot 7 - 5 \cdot 8$.

Solution:

We use the order of operations to simplify this expression. First we multiply, and then subtract from left to right.

Multiply first.	$3 \cdot 7 - 4 \cdot 7 - 5 \cdot 8$
Subtract from left to right.	$21 - 28 - 40$
Subtract.	$-7 - 40$
	-47

Note:
Try It
Exercise:

Problem: Simplify the expression: $6 \cdot 2 - 9 \cdot 1 - 8 \cdot 9.$
Solution: −69

Summary

- **Subtraction of Integers**

5 – 3	–5 – (–3)
2	–2
2 positives	2 negatives
When there would be enough counters of the color to take away, subtract.	
–5 – 3	5 – (–3)
–8	8
5 negatives, want to subtract 3 positives	5 positives, want to subtract 3 negatives
need neutral pairs	need neutral pairs

- **Subtraction Property**
  - $a - b = a + (-b)$
  - $a - (-\text{b}) = a + b$

Homework

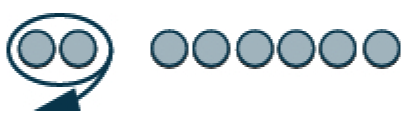
Model Subtraction of Integers

In the following exercises, model each expression and simplify.

Exercise:

Problem: $8 - 2$

Solution:



8 - 2 = 6

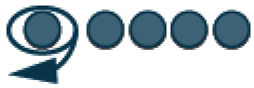
Exercise:

Problem: $9 - 3$

Exercise:

Problem: $-5 - (-1)$

Solution:



-5 - (-1) = -4

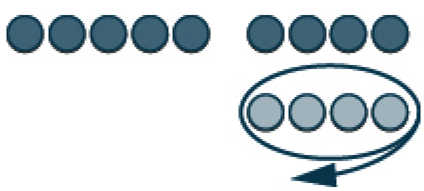
Exercise:

Problem: $-6-(-4)$

Exercise:

Problem: $-5-4$

Solution:



-5 - 4 = -9

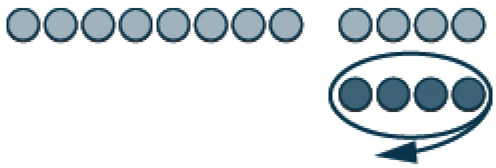
Exercise:

Problem: $-7-2$

Exercise:

Problem: $8-(-4)$

Solution:



8 - (-4) = 12

Exercise:

Problem: $7-(-3)$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

Exercise:

Problem:

$$15-6$$
$$15+(-6)$$

Solution:

$$9$$
$$9$$

Exercise:

Problem:

$$12-9$$
$$12+(-9)$$

Exercise:

Problem:

$44 - 28$
$44 + (-28)$

Solution:

16
16

Exercise:

Problem:

$35 - 16$
$35 + (-16)$

Exercise:

Problem:

$8 - (-9)$
$8 + 9$

Solution:

17
17

Exercise:

Problem:

$4 - (-4)$
$4 + 4$

Exercise:

Problem:

$$27 - (-18)$$
$$27 + 18$$

Solution:

45
45

Exercise:

Problem:

$$46 - (-37)$$
$$46 + 37$$

In the following exercises, simplify each expression. Be sure you can do these without a calculator.

Exercise:

Problem: $15 - (-12)$

Solution:

27

Exercise:

Problem: $14 - (-11)$

Exercise:

Problem: $10 - (-19)$

Solution:

29

Exercise:

Problem: $11 - (-18)$

Exercise:

Problem: $48 - 87$

Solution:

-39

Exercise:

Problem: $45 - 69$

Exercise:

Problem: $31 - 79$

Solution:

-48

Exercise:

Problem: $39 - 81$

Exercise:

Problem: $-31 - 11$

Solution:

-42

Exercise:

Problem: $-32-18$

Exercise:

Problem: $-17-42$

Solution:

-59

Exercise:

Problem: $-19-46$

Exercise:

Problem: $-103-(-52)$

Solution:

-51

Exercise:

Problem: $-105-(-68)$

Exercise:

Problem: $-45-(-54)$

Solution:

9

Exercise:

Problem: $-58-(-67)$

Exercise:

Problem: $8-3-7$

Solution:

-2

Exercise:

Problem: $9-6-5$

Exercise:

Problem: $-5-4+7$

Solution:

-2

Exercise:

Problem: $-3-8+4$

Exercise:

Problem: $-14-(-27)+9$

Solution:

22

Exercise:

Problem: $-15-(-28)+5$

Exercise:

Problem: $71+(-10)-8$

Solution:

53

Exercise:

Problem: $64 + (-17) - 9$

Exercise:

Problem: $-16 - (-4 + 1) - 7$

Solution:

-20

Exercise:

Problem: $-15 - (-6 + 4) - 3$

Exercise:

Problem: $(2 - 7) - (3 - 8)$

Solution:

0

Exercise:

Problem: $(1 - 8) - (2 - 9)$

Exercise:

Problem: $-(6 - 8) - (2 - 4)$

Solution:

4

Exercise:

Problem: $-(4-5)-(7-8)$

Exercise:

Problem: $25-[10-(3-12)]$

Solution:

6

Exercise:

Problem: $32-[5-(15-20)]$

Exercise:

Problem: $6\cdot 3-4\cdot 3-7\cdot 2$

Solution:

-8

Exercise:

Problem: $5\cdot 7-8\cdot 2-4\cdot 9$

Exercise:

Problem: 5^2-6^2

Solution:

-11

Exercise:

Problem: $6^2 - 7^2$

Multiplying and Dividing Integers

Learning Objectives

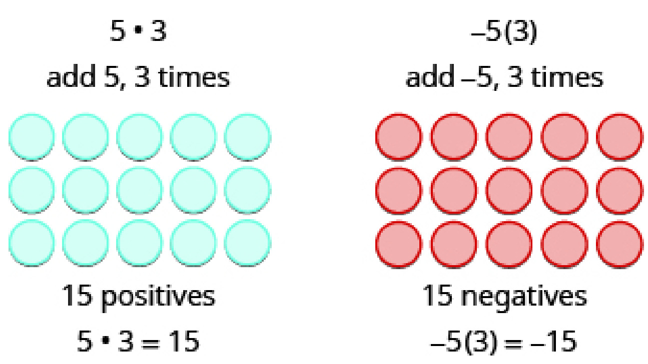
By the end of this lesson, you will be able to:

- Multiply integers
- Divide integers
- Simplify expressions with integers

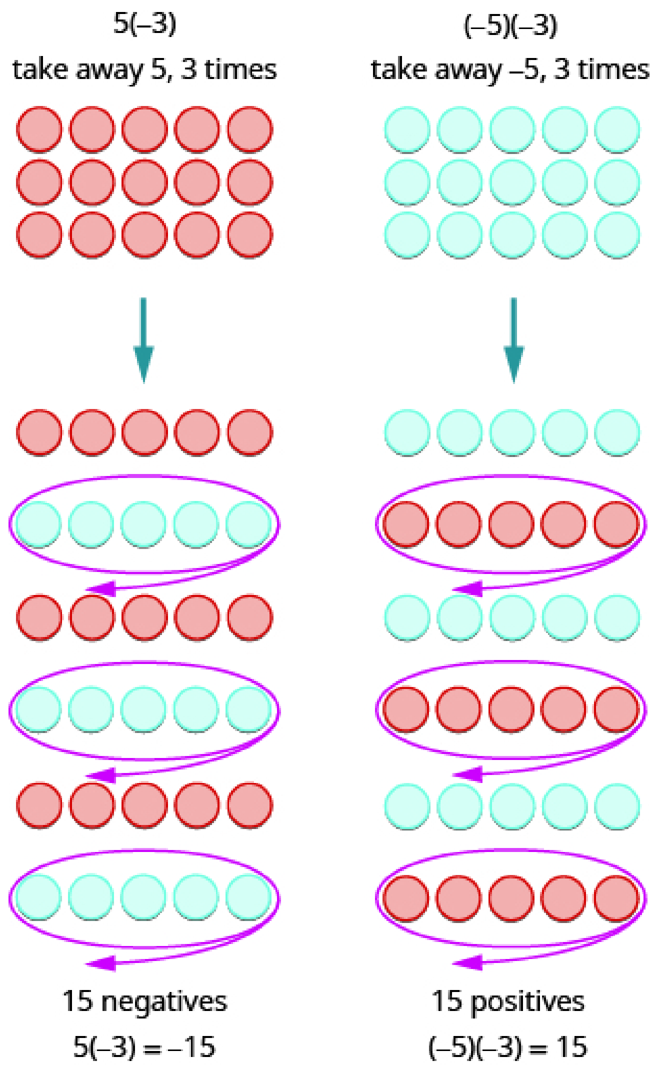
Multiply Integers

Since multiplication is mathematical shorthand for repeated addition, our counter model can easily be applied to show multiplication of integers. Let’s look at this concrete model to see what patterns we notice. We will use the same examples that we used for addition and subtraction.

We remember that $a \cdot b$ means add $a,\ b$ times. Here, we are using the model shown in [link] just to help us discover the pattern.



Now consider what it means to multiply 5 by -3. It means subtract $5, 3$ times. Looking at subtraction as *taking away*, it means to take away $5, 3$ times. But there is nothing to take away, so we start by adding neutral pairs as shown in [link].



In both cases, we started with $\mathbf{15}$ neutral pairs. In the case on the left, we took away $\mathbf{5}, \mathbf{3}$ times and the result was $-\mathbf{15}$. To multiply $(-5)\,(-3)$, we took away $-\mathbf{5}, \mathbf{3}$ times and the result was $\mathbf{15}$. So we found that

Equation:

$$\begin{array}{ll} 5 \cdot 3 = 15 & -5(3) = -15 \\ 5(-3) = -15 & (-5)(-3) = 15 \end{array}$$

Notice that for multiplication of two signed numbers, when the signs are the same, the product is positive, and when the signs are different, the product is negative.

Note:
Multiplication of Signed Numbers
The sign of the product of two numbers depends on their signs.

Same signs	Product
•Two positives •Two negatives	Positive Positive

Different signs	Product
•Positive • negative •Negative • positive	Negative Negative

Example:
Exercise:

Problem: Multiply each of the following:

$-9 \cdot 3$
$-2(-5)$
$4(-8)$
$7 \cdot 6$

Solution:

a	
	$-9 \cdot 3$

Multiply, noting that the signs are different and so the product is negative.	-27

b	
	$-2(-5)$
Multiply, noting that the signs are the same and so the product is positive.	10

c	
	$4(-8)$
Multiply, noting that the signs are different and so the product is negative.	-32

d	
	$7 \cdot 6$
The signs are the same, so the product is positive.	42

Note:
Try It
Exercise:

Problem: Multiply:

$-6 \cdot 8$
$-4(-7)$
$9(-7)$
$5 \cdot 12$

Solution:

−48
28
−63
60

Note:
Try It
Exercise:

Problem: Multiply:

a. $-8 \cdot 7$
b. $-6(-9)$
c. $7(-4)$
d. $3 \cdot 13$

Solution:

-56
54
-28
39

Divide Integers

Division is the inverse operation of multiplication. So, $15 \div 3 = 5$ because $5 \cdot 3 = 15$ In words, this expression says that $\mathbf{15}$ can be divided into $\mathbf{3}$ groups of $\mathbf{5}$ each because adding five three times gives $\mathbf{15}$. If we look at some examples of multiplying integers, we might figure out the rules for dividing integers.

Equation:

$$5 \cdot 3 = 15 \text{ so } 15 \div 3 = 5 \qquad -5(3) = -15 \text{ so } -15 \div 3 = -5$$
$$(-5)(-3) = 15 \text{ so } 15 \div (-3) = -5 \qquad 5(-3) = -15 \text{ so } -15 \div -3 = 5$$

Division of signed numbers follows the same rules as multiplication. When the signs are the same, the quotient is positive, and when the signs are different, the quotient is negative.

Note:
Division of Signed Numbers
The sign of the quotient of two numbers depends on their signs.

Same signs	Quotient
•Two positives •Two negatives	Positive Positive

Different signs	Quotient
•Positive ÷ negative •Negative ÷ positive	Negative Negative

Remember, you can always check the answer to a division problem by multiplying.

Example:
Exercise:

Problem: Divide each of the following:

$-27 \div 3$
$-100 \div (-4)$

Solution:

a	
	$-27 \div 3$
Divide, noting that the signs are different and so the product is negative.	-9

b	
	$-100 \div (-4)$
Divide, noting that the signs are the same and so the product is positive.	25

Note:
Try It
Exercise:

Problem: Divide:

$-42 \div 6$
$-117 \div (-3)$

Solution:

−7
39

Note:
Try It
Exercise:

Problem: Divide:

$-63 \div 7$
$-115 \div (-5)$

Solution:

−9
23

Simplify Expressions with Integers

Now we’ll simplify expressions that use all four operations–addition, subtraction, multiplication, and division–with integers. Remember to follow the order of operations.

Example:
Exercise:

Problem: Simplify: $7(-2) + 4(-7) - 6$.

Solution:

We use the order of operations. Multiply first and then add and subtract from left to right.

	$7(-2) + 4(-7) - 6$
Multiply first.	$-14 + (-28) - 6$
Add.	$-42 - 6$
Subtract.	-48

Note:
Try It
Exercise:

Problem: Simplify:

$$8(-3) + 5(-7) - 4$$

Solution:

-63

Note:
Try It
Exercise:

Problem: Simplify:

$$9(-3) + 7(-8) - 1$$

Solution:

-84

Example:
Exercise:

Problem: Simplify:

$$(-2)^4$$

-2^4

Solution:

The exponent tells how many times to multiply the base.

a. The exponent is 4 and the base is -2. We raise -2 to the fourth power.

	$(-2)^4$
Write in expanded form.	$(-2)(-2)(-2)(-2)$
Multiply.	$4(-2)(-2)$
Multiply.	$-8(-2)$
Multiply.	16

b. The exponent is 4 and the base is 2. We raise 2 to the fourth power and then take the opposite.

	-2^4
Write in expanded form.	$-(2 \cdot 2 \cdot 2 \cdot 2)$
Multiply.	$-(4 \cdot 2 \cdot 2)$
Multiply.	$-(8 \cdot 2)$

Multiply.	-16

Note:
Try It
Exercise:

Problem: Simplify:

$(-3)^4$
-3^4

Solution:

81
−81

Example:
Exercise:

Problem: Simplify: $12 - 3(9 - 12)$.

Solution:

According to the order of operations, we simplify inside parentheses first. Then we will multiply and finally we will subtract.

	$12 - 3(9 - 12)$
Subtract the parentheses first.	$12 - 3(-3)$
Multiply.	$12 - (-9)$
Subtract.	21

Note:
Try It
Exercise:

Problem: Simplify:

$17 - 4(8 - 11)$

Solution:

29

Example:
Exercise:

Problem: Simplify: $8(-9) \div (-2)^3$.

Solution:

We simplify the exponent first, then multiply and divide.

	$8(-9) \div (-2)^3$
Simplify the exponent.	$8(-9) \div (-8)$
Multiply.	$-72 \div (-8)$
Divide.	9

Note:
Try It
Exercise:

Problem: Simplify:

$12(-9) \div (-3)^3$

Solution:

4

Note:
Try It
Exercise:

Problem: Simplify:

$18(-4) \div (-2)^3$

Solution:

9

Example:
Exercise:

Problem: Simplify: $-30 \div 2 + (-3)(-7)$.

Solution:

First we will multiply and divide from left to right. Then we will add.

	$-30 \div 2 + (-3)(-7)$
Divide.	$-15 + (-3)(-7)$
Multiply.	$-15 + 21$
Add.	6

Note:
Try It
Exercise:

Problem: Simplify:

$-27 \div 3 + (-5)(-6)$

Solution:

21

Note:
Try It
Exercise:

Problem: Simplify:

$$-32 \div 4 + (-2)(-7)$$

Solution:

6

Summary

Multiplication and Division of Signed Numbers

Same signs	Product
•Two positives •Two negatives	Positive Positive

Different signs	Product
•Positive • negative •Negative • positive	Negative Negative

Same signs	Quotient
•Two positives •Two negatives	Positive Positive

Different signs	Quotient
•Positive ÷ negative •Negative ÷ positive	Negative Negative

Homework

Multiply Integers

In the following exercises, multiply each pair of integers.

Exercise:

Problem: $-4 \cdot 8$

Solution:

-32

Exercise:

Problem: $-3 \cdot 9$

Exercise:

Problem: $-5(7)$

Solution:

-35

Exercise:

Problem: $-8(6)$

Exercise:

Problem: $-18(-2)$

Solution:

36

Exercise:

Problem: $-10(-6)$

Exercise:

Problem: $9(-7)$

Solution:

-63

Exercise:

Problem: $13(-5)$

Exercise:

Problem: $-1 \cdot 6$

Solution:

-6

Exercise:

Problem: $-1 \cdot 3$

Exercise:

Problem: $-1(-14)$

Solution:

14

Exercise:

Problem: $-1(-19)$

Divide Integers

In the following exercises, divide.

Exercise:

Problem: $-24 \div 6$

Solution:

-4

Exercise:

Problem: $-28 \div 7$

Exercise:

Problem: $56 \div (-7)$

Solution:

-8

Exercise:

Problem: $35 \div (-7)$

Exercise:

Problem: $-52 \div (-4)$

Solution:

13

Exercise:

Problem: $-84 \div (-6)$

Exercise:

Problem: $-180 \div 15$

Solution:

-12

Exercise:

Problem: $-192 \div 12$

Exercise:

Problem: $49 \div (-1)$

Solution:

-49

Exercise:

Problem: $62 \div (-1)$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $5(-6) + 7(-2) - 3$

Solution:

-47

Exercise:

Problem: $8(-4) + 5(-4) - 6$

Exercise:

Problem: $-8(-2)-3(-9)$

Solution:

43

Exercise:

Problem: $-7(-4)-5(-3)$

Exercise:

Problem: $(-5)^3$

Solution:

-125

Exercise:

Problem: $(-4)^3$

Exercise:

Problem: $(-2)^6$

Solution:

64

Exercise:

Problem: $(-3)^5$

Exercise:

Problem: -4^2

Solution:

-16

Exercise:

Problem: -6^2

Exercise:

Problem: $-3(-5)(6)$

Solution:

90

Exercise:

Problem: $-4(-6)(3)$

Exercise:

Problem: $-4 \cdot 2 \cdot 11$

Solution:

-88

Exercise:

Problem: $-5 \cdot 3 \cdot 10$

Exercise:

Problem: $(8-11)(9-12)$

Solution:

9

Exercise:

Problem: $(6-11)(8-13)$

Exercise:

Problem: $26 - 3(2 - 7)$

Solution:

41

Exercise:

Problem: $23 - 2(4 - 6)$

Exercise:

Problem: $-10(-4) \div (-8)$

Solution:

-5

Exercise:

Problem: $-8(-6) \div (-4)$

Exercise:

Problem: $65 \div (-5) + (-28) \div (-7)$

Solution:

-9

Exercise:

Problem: $52 \div (-4) + (-32) \div (-8)$

Exercise:

Problem: $9 - 2[3 - 8(-2)]$

Solution:

-29

Exercise:

Problem: $11 - 3[7 - 4(-2)]$

Exercise:

Problem: $(-3)^2 - 24 \div (8 - 2)$

Solution:

5

Exercise:

Problem: $(-4)^2 - 32 \div (12 - 4)$

Meanings of Operations

Learning Objectives

By the end of this lesson, you will be able to:

- Understand the meanings of the operations of addition, subtraction, multiplication, and division.
- Use the meanings of operations and key words/phrases to write mathematical calculations and solve problems.

The following table summarizes the four main mathematical operations, their meanings and key words:

Operation	Name of Result	Meaning	Some Key Words or Phrases
Addition	sum/total	to combine or find the total	sum combine total plus more than increase by
Subtraction	difference	to reduce or compare	difference minus reduce by decrease by less than
Multiplication	product	repeated addition, or to scale up or down	product (fraction) of (percent) of times twice, double
Division	quotient	find the count of groups or sections	quotient per divide ratio

For each of the following examples, we will use the meanings and key words/phrases of the different operations to write a mathematical

calculation and then use it to answer the question.

Example:
Exercise:

Problem:

What is the total weight of two boxes if one box weighs 3.45 lb. and the other box weighs 7.86 lb.?

Solution:

In this problem, we need to **combine** weights, so the operation is **addition**. Also, notice the key word "total".

Therefore, the calculation is: 3.45 + 7.86

The result is 11.31, so the total weight is 11.31 lb. (Always include a label of units on your answers.)

Note:
Try It
Exercise:

Problem:

A drug is mixed with $\frac{3}{4}$ oz. of drug A and $\frac{1}{2}$ oz. of drug B. What is the total weight?

Solution:

The calculation is: $\frac{3}{4}+\frac{1}{2}$

The answer is: $1\frac{1}{4}$ oz.

Example:
Exercise:

Problem:

A bottle contains $\frac{5}{8}$ gram of a drug and $\frac{1}{4}$ gram was given out yesterday. How many grams remain in the bottle?

Solution:

The meaning is **to reduce** so the operation is **subtraction** and the calculation is: $\frac{5}{8} - \frac{1}{4}$

The answer is $\frac{3}{8}$ gram.

Note:
Try It
Exercise:

Problem:

An infant weighed 8 lb 14 oz at birth and later the weight was measured at 8 lb 11 oz. How much weight did the infant lose?

Solution:

The calculation is: 8 lb. 14 oz. – 8 lb. 11 oz.

The answer is: 3 oz.

Example:
Exercise:

Problem:

There are 75 tablets in a bottle and each tab is 0.25 grams. How many grams of medication are in the bottle?

Solution:

For this one, you can think of it as **repeated addition** of adding 0.25 grams 75 times. Of course, this is the same as **multiplication**.

The calculation is: (0.25)(75).

The answer is 18.75 grams.

Note:
Try It
Exercise:

Problem:

There are 25 people in a clinical trial and they each receive 14 tablets a week. How many tablets are dispensed in a week?

Solution:

This is a scaling up due to repeated addition, so the calculation is 25(14).

The answer is 350 tablets.

Example:
Exercise:

Problem:

A vial contains 1600 milligrams and a dose is 20 milligrams per day. How many days' supply are in the vial?

Solution:

This is asking **how many sections**, of 20 mg each, are in the vial. Each section is a day’s supply. So the operation is **division**.

The calculation is 1600/20.

The answer is 80 days.

Note:
Try It
Exercise:

Problem:

A tablet contains $\frac{3}{4}$ gram of drug. The dose for this drug is $\frac{3}{8}$ gram. How many doses are in the tablet?

Solution:

The calculation is $\frac{3}{4} \div \frac{3}{8}$

The answer is 2 grams.

Homework

For the following exercises, determine the operation and write the calculation needed to solve the problem. Then, give the answer, including units.

Exercise:

Problem:

How many mg of drug are needed to make 12 tablets if each tablet is to have $\frac{3}{4}$ mg of drug?

Solution:

The calculation is $\frac{3}{4} \times 12$. The answer is 9 mg.

Exercise:

Problem:

A tablet contains 0.75 gram of drug. A single dose for this drug is 0.25 gram. How many doses are in the tablet?

Exercise:

Problem:

A container holds $\frac{7}{8}$ gallon of a solvent. If $\frac{1}{2}$ gallon is used, how much is left in the container?

Solution:

The calculation is $\frac{7}{8} - \frac{1}{2}$. The answer is $\frac{3}{8}$ gallon.

Exercise:

Problem:

A patient was given a part of a tablet. A whole tablet contains 6 mg of drug. If the patient was given $\frac{1}{4}$ of the tablet, how many mg did the patient receive?

Exercise:

Problem:

A lab technician is weighing a denture in ounces and measures it as less than one full ounce. The technician records the weight as $\frac{15}{16}$ of an ounce. This denture weighs too much and $\frac{1}{5}$ of the weight needs to be trimmed off. How much weight is to be removed?

Solution:

The calculation is: $\frac{1}{5} \times \frac{15}{16}$. The answer is $\frac{3}{16}$ oz.

Exercise:

Problem:

How many tablets can be made from 20 mg of drug if each tablet weighs 1.5 mg?

Exercise:

Problem:

A drug is administered in a 2.5 mL vial. How many vials are needed to administer 60 mL of the drug?

Solution:

The calculation is 60/2.5. The answer is 24 vials.

Exercise:

Problem:

Six capsules of drug are to be administered every day. How many days' supply would 48 capsules be?

Exercise:

Problem:

A patient is prescribed 2 drugs. One drug weighs 250 mg for a dose and the other drug weighs 125 mg for a dose. How many mg total is the patient prescribed?

Solution:

The calculation is 250 + 125. The answer is 375 mg.

Exercise:

Problem:

What is the total volume of a medication if $\frac{5}{6}$ oz of drug A is mixed with $1\frac{2}{3}$ oz of drug B?

Exercise:

Problem:

A lab technician has to perform a test on a third of a sample. The sample weighs $\frac{7}{8}$ of an ounce. How many ounces will she be testing?

Solution:

The calculation is $\frac{1}{3} \times \frac{7}{8}$. The answer is $\frac{7}{24}$ oz.

Exercise:

Problem:

Each tablet contains 120 milligrams of a drug. If a pharmacist has a total of 3000 milligrams of the drug, how many tablets can be made?

Evaluating Formulas

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Review the order of operations.
- Evaluate formulas.
- Solve application problems, by evaluating given formulas.

The Order of Operations

Recall that **PEMDAS** is an acronym for the correct order of operations.

- First do any operation within parentheses (P). This also includes above or below a fraction bar, under a radical sign, and inside absolute value signs.
- Next, perform all exponents (E).
- Then, perform any multiplication or division **as they appear from left to right** (MD).
- Lastly, perform any subtraction or addition **as they appear from left to right** (AS).

Note: Note that just because M comes before D, in PEMDAS, does NOT mean that you perform all multiplications before doing any division. You must perform whichever appears first as you move from left to right. The same thing is true for addition and subtraction. If necessary, review this concept in The Order of Operations.

Example:
Exercise:

Problem:

Evaluate by following the correct order of operations:
$6 + 12 \div 3 \cdot 2 - (4 + 3) + 2^3$

Solution:

- First perform the addition inside the parentheses, which will give you $6 + 12 \div 3 \cdot 2 - 7 + 2^3$.
- Next, perform the exponent to get $6 + 12 \div 3 \cdot 2 - 7 + 8$.
- Then, divide 12 by 3, since it is the first multiplication or division we see as we read from left to right. This will give you $6 + 4 \cdot 2 - 7 + 8$.
- The multiplication of 4 and 2 should be performed next to give you $6 + 8 - 7 + 8$.
- We now need to perform all additions and subtractions from left to right, which means the 6 and 8 should be added next, which gives us $14 - 7 + 8$.
- Subtracting 7 from 14 would come next, giving us $7 + 8$, which equals 15. So our final answer is 15.

Evaluating a Formula

To evaluate a formula, replace all variables (letters) with their given values and then simplify, using the correct order of operations.

Example:

Exercise:

Problem:

Evaluate the formula $a = \frac{A}{bc}$ given that $A = 20$, $b = 2$, and $c = 5$.

Solution:

First, we replace all the variables with their given values, which will give us $a = \frac{20}{(2)(5)}$.

Next, we perform the multiplication in the denominator to get $a = \frac{20}{10}$.

Now, we perform the division to get our final answer, which is $a = 2$.

Example:
Exercise:

Problem:

Evaluate the formula for the volume of a box: $V = \mathrm{LWH}$, given that $L = 14.7$, $W = 5.2$, and $H = 2$.

Solution:

First, we replace all the variables with their given values, which will give us

$$V = (14.7)(5.2)(2)$$

Next, we perform the multiplication to get $V = 152.88$.

Note: We'll learn what to do when there are units involved in a later section.

Applications with Formulas

Slope Formula

Some formulas may use subscripts (a small number or letter written slightly below and to the right of the variable). This formula is using words with subscripts. It could also use variables instead, as shown. To find the slope of a line, we use the formula

$$m = \frac{\text{output}_2 - \text{output}_1}{\text{input}_2 - \text{input}_1} \quad \text{or} \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example:
Exercise:

Problem:

Use the slope formula to solve for m and simplify to a unit rate. Include labels in the rate. Let y_2 = 2.1 mL, y_1 = 1.6 mL, x_2 = 0.2 g, and x_1 = 0.1 g.

Solution:

First we substitute the values in for all the variables that are given. Then, we simplify by following the order of operations, which will give us

$m = \frac{2.1-1.6}{0.2-0.1} = \frac{0.5}{0.1} = \frac{5}{1} = 5\text{ mL/g}.$

Notice how we label the answer with the appropriate units. This is a very important step.

Note:
Try It
Exercise:

Problem:

Use the slope formula to solve for m and simplify to a unit rate. Include labels in the rate. Let y_2 = 25 cents, y_1 = 10 cents, x_2 = 2 oz., and x_1 = 1.5 oz.

Solution:

$m = 30\ \text{cents/oz.}$

Exercise Movement Example

There are many formulas for Metabolic Calculations. They use very specific numbers and units.

Here is a formula for the Metabolism for walking:
$\text{V0}\,2 = 0.1S + 1.8SG + 3.5$, where VO2 represents the maximum amount of oxygen the body can use during a specified period of exercise and is measured in mL/(kg*min).

- It is made up of 3 terms to add. Notice the constants of 0.1, 1.8, and 3.5.
- S is the speed in meters/min (we will do more with this in conversion lessons)
- G is the grade of incline given as a percent and calculated as a decimal. So 3% would be 0.03.

- The constant 3.5 is for 1 MET (measured in mL/(kg*min)), which is a measure of the rate at which the body uses energy, called “metabolic equivalent”.

Example:
Exercise:

Problem:

Find the VO2 for walking a speed of 94.1 meters per minute (which is equivalent to 3.5 mph) up a 5% grade.

Solution:

Plug in the values for S and G and simplify:

$$\mathrm{VO2} = 0.1\,(94.1) + 1.8\,(94.1)\,(0.05) + 3.5 \approx 21.3\ \mathrm{mL}/(\mathrm{kg} * \mathrm{min})$$

Note:
Try It
Exercise:

Problem:

Find the VO2 for walking a speed of 50 meters per minute up a 7.5% grade.

Solution:

$\mathrm{VO2} = 15.25\ \mathrm{mL}/(\mathrm{kg}{*}\mathrm{min})$

Here is a formula for the Metabolism for running:
$\mathrm{V0\,2} = 0.2S + 0.9SG + 3.5$, where VO2 represents the maximum amount of oxygen the body can use during a specified period of exercise and is measured in mL/(kg*min).

- Notice the constants of 0.2, 0.9, and 3.5.
- S is the speed in meters/min (we will do more with this in conversion lessons)
- G is the grade of incline given as a percent and calculated as a decimal. So 3% would be 0.03.
- The constant 3.5 is for 1 MET (measured in mL/(kg*min)), which is a measure of the rate at which the body uses energy, called “metabolic equivalent”.

Example:
Exercise:

Problem:

Find the VO2 for running a speed of 174.7 meters per minute (which is equivalent to 6.5 mph) on level ground.

Solution:

Plug in the values for S and G and simplify. Note that level ground means the grade is 0%.

$$\mathrm{VO2} = 0.2(174.7) + 0.9(174.7)(0) + 3.5 \approx 38.4\ \mathrm{mL/(kg * min)}$$

Note:
Try It
Exercise:

Problem:

Find the VO2 for running a speed of 225 meters per minute up a 2% grade.

Solution:

$VO2 = 52.55 \text{ mL/(kg*min)}$

Temperature Scales

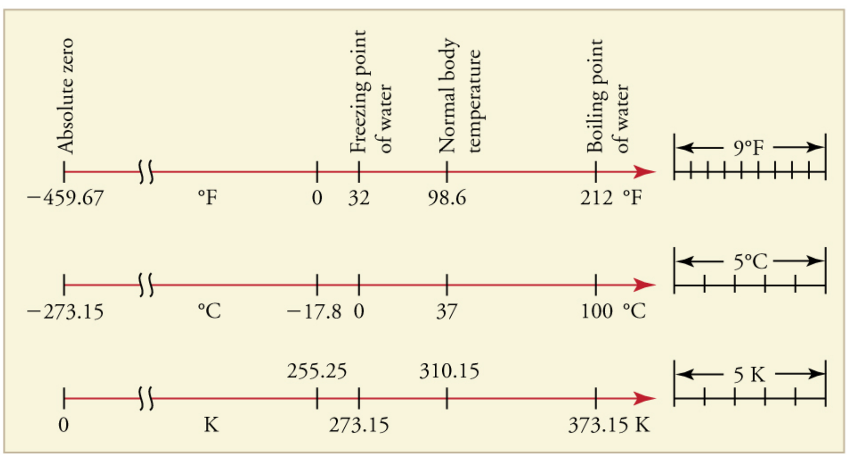
The concept of temperature has evolved from the common concepts of hot and cold. Human perception of what feels hot or cold is a relative one. For example, if you place one hand in hot water and the other in cold water, and then place both hands in tepid water, the tepid water will feel cool to the hand that was in hot water, and warm to the one that was in cold water. The scientific definition of temperature is less ambiguous than your senses of hot and cold. **Temperature** is operationally defined to be what we measure with a thermometer.

Thermometers are used to measure temperature according to well-defined scales of measurement, which use pre-defined reference points to help compare quantities. The three most common temperature scales are the Fahrenheit, Celsius, and Kelvin scales. A temperature scale can be created by identifying two easily reproducible temperatures. The freezing and boiling temperatures of water at standard atmospheric pressure are commonly used.

The **Celsius** scale (which replaced the slightly different *centigrade* scale) has the freezing point of water at $0^{o}C$ and the boiling point at $100^{o}C$. Its unit is the **degree Celsius**(^{o}C). On the **Fahrenheit** scale (still the most frequently used in the United States), the freezing point of water is at $32^{o}F$ and the boiling point is at $212^{o}F$. The unit of temperature on this scale is the **degree Fahrenheit**(^{o}F). Note that a temperature difference of one degree Celsius is greater than a temperature difference of one degree

Fahrenheit. Only 100 Celsius degrees span the same range as 180 Fahrenheit degrees, thus one degree on the Celsius scale is 1.8 times larger than one degree on the Fahrenheit scale $180/100 = 9/5$.

The **Kelvin** scale is the temperature scale that is commonly used in science. It is an *absolute temperature* scale defined to have 0 K at the lowest possible temperature, called **absolute zero**. The official temperature unit on this scale is the *kelvin*, which is abbreviated K, and is not accompanied by a degree sign. The freezing and boiling points of water are 273.15 K and 373.15 K, respectively. Thus, the magnitude of temperature differences is the same in units of kelvins and degrees Celsius. Unlike other temperature scales, the Kelvin scale is an absolute scale. It is used extensively in scientific work because a number of physical quantities, such as the volume of an ideal gas, are directly related to absolute temperature. The kelvin is the SI unit used in scientific work.



Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown.

The relationships between the three common temperature scales is shown in [link]. Temperatures on these scales can be converted using the equations in

[link].

To convert from . . .	Use this formula . . .
Celsius to Fahrenheit	$F = \frac{9}{5}C + 32$
Fahrenheit to Celsius	$C = \frac{5}{9}(F - 32)$
Celsius to Kelvin	$K = C + 273.15$
Kelvin to Celsius	$C = K - 273.15$
Fahrenheit to Kelvin	$K = \frac{5}{9}(F - 32) + 273.15$
Kelvin to Fahrenheit	$F = \frac{9}{5}(K - 273.15) + 32$

Temperature Conversions

Notice that the conversions between Fahrenheit and Kelvin look quite complicated. In fact, they are simple combinations of the conversions between Fahrenheit and Celsius, and the conversions between Celsius and Kelvin.

Example:

Converting between Temperature Scales: Room Temperature

"Room temperature" is generally defined to be $25^{\circ}C$. (a) What is room temperature in $^{\circ}F$? (b) What is it in $^{\circ}K$?

Strategy

To answer these questions, all we need to do is choose the correct conversion equations and plug in the known values.

Solution for (a)

1. Choose the right equation. To convert from $^{\circ}C$ to $^{\circ}F$, use the equation

Equation:

$$F = \frac{9}{5}C + 32$$

2. Plug the known value into the equation, simplify, and include units on the answer:

Equation:

$$F = \frac{9}{5}\left(25\right) + 32 = 45 + 32 = 77^{\circ}\mathrm{F}$$

Solution for (b)

1. Choose the right equation. To convert from $^{\circ}C$ to $^{\circ}K$, use the equation

Equation:

$$K = C + 273.15$$

2. Plug the known value into the equation, simplify, and include units on the answer:

Equation:

$$K = (25) + 273.15 = 298.15^{\circ}\mathrm{K}$$

Note:

Try It

Exercise:

Problem: Convert 59 degrees Fahrenheit to degrees Celsius.

Solution:

$15^\circ\mathrm{C}$

Homework

Exercise:

Problem:

Find m, using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, when $x_2 = 8$ g, $x_1 = 5$ g, $y_2 = 3$ mL, and $y_1 = 12$ mL.

Solution:

$m = -3$ mL/g.

Exercise:

Problem:

Find m, using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, when $x_2 = 9$ g, $x_1 = 4$ g, $y_2 = 10$ mL, and $y_1 = 2$ mL.

Exercise:

Problem:

Find the VO2 for a woman running 147.8 meters per minute on a 7.5% incline.

Solution:

43 mL/(kg*min)

Exercise:

Problem:

Find the VO2 for a man walking 3 miles per hour on a 10% incline.

Exercise:

Problem: Convert 30°C to degrees Fahrenheit.

Solution:

86°F

Exercise:

Problem: Convert 98.6°F to degrees Celsius.

Exercise:

Problem:

What is the Fahrenheit temperature of a person with a 39.0°C fever?

Solution:

102°F

Exercise:

Problem:

Frost damage to most plants occurs at temperatures of 28.0°F or lower. What is this temperature on the Kelvin scale?

Exercise:

Problem:

To conserve energy, room temperatures are kept at 68.0°F in the winter and 78.0°F in the summer. What are these temperatures on the Celsius scale?

Solution:

$20.0^\circ\mathrm{C}$ and $25.6^\circ\mathrm{C}$

Exercise:

Problem:

A tungsten light bulb filament may operate at $2900^\circ\mathrm{K}$. What is its Fahrenheit temperature? What is this on the Celsius scale?

Exercise:

Problem:

The surface temperature of the Sun is about $5750^\circ\mathrm{K}$. What is this temperature on the Fahrenheit scale?

Solution:

$9890^\circ\mathrm{F}$

Exercise:

Problem:

One of the hottest temperatures ever recorded on the surface of Earth was $134^\circ\mathrm{F}$ in Death Valley, CA. What is this temperature in Celsius degrees? What is this temperature in Kelvin?

Solving Linear Equations in One Variable
An introduction to solving basic linear equations in one variable.

Learning Objectives

By the end of this lesson, you will be able to

- Apply the properties of equality to solve basic linear equations in one or two steps.
- Combine like variable terms to solve linear equations of the form $ax + b = cx + d$ in multiple steps.

Solving Linear Equations in One Variable

A **linear equation in one variable** is an equation of the form $ax + b = c$, where a is called a **coefficient**, x is called the **variable**, and a, b, and c are all **constants** (a cannot be zero).

When the coefficients of linear equations are numbers other than nice easy integers, guessing at solutions becomes an unreasonable prospect. Let's look at an algebraic technique for solving these types of equations by first looking at the **properties of equality**.

Properties of Equality

Given algebraic expressions A and B where c is a real number:

Property 1: Addition Property of Equality

$$\text{If } A = B \text{ then } A + c = B + c$$

Property 2: Subtraction Property of Equality

$$\text{If } A = B \text{ then } A - c = B - c$$

Property 3: Multiplication Property of Equality

$$\text{If } A = B \text{ and } c \neq 0 \text{ then c} A = cB$$

Property 4: Division Property of Equality

$$\text{If } A = B \text{ and } c \neq 0 \text{ then } \frac{A}{c} = \frac{B}{c}$$

Note: Multiplying or dividing both sides of an equation by zero is carefully avoided. Dividing by zero is undefined and multiplying both sides by zero will result in an equation 0=0.

To summarize, the equality is retained if we add, subtract, multiply and divide both sides of an equation by any nonzero real number. The central technique for solving linear equations involves applying these properties in order to isolate the variable on one side of the equation.

Example:
Use the properties of equality to solve: $x + 3 = -5$

$$\begin{aligned} x + 3 &= -5 \\ x + 3 - 3 &= -5 - 3 && \textit{Subtract 3 on both sides.} \\ x &= -8 && \textit{Simplify} \end{aligned}$$

The solution set is $\{-8\}$.

Example:
Use the properties of equality to solve: $-5x = -35$

$$\begin{aligned} -5x &= -35 \\ \frac{-5x}{-5} &= \frac{-35}{-5} && \textit{Divide both sides by -5.} \\ x &= 7 && \textit{Simplify} \end{aligned}$$

The solution set is $\{7\}$.

Two other important properties are:

Property 5: Symmetric Property

$$\text{If } A = B \text{ then } B = A.$$

When solving, we often see $2 = x$, but that is equivalent to $x = 2$.

Property 6: Transitive Property

$$\text{If } A = B \text{ and } B = C \text{ then } A = C.$$

Isolating the Variable

The idea behind solving in algebra is to isolate the variable. If given a linear equation of the form $ax + b = c$, we can solve it in two steps. First use the equality property of addition or subtraction to isolate the variable term. Next isolate the variable by using the equality property of multiplication or division. The property choice depends on the given operation, we choose to apply the

opposite property of the given operation. For example, if given a term plus three we would first choose to subtract three on both sides of the equation. If given two times the variable then we would choose to divide both sides by two.

Example:
Solve $2x + 3 = 13$.

$$\begin{array}{rcll} 2x+3 & = & 13 & \\ 2x+3-3 & = & 13-3 & \textit{Subtract 3 on both sides.} \\ 2x & = & 10 & \\ \frac{2x}{2} & = & \frac{10}{2} & \textit{Divide both sides by 2.} \\ x & = & 5 & \end{array}$$

The solution set is $\{5\}$.

Example:
Solve $-3x - 2 = 9$.

$$\begin{array}{rcll} -3x-2 & = & 9 & \\ -3x-2+2 & = & 9+2 & \textit{Add 2 to both sides.} \\ -3x & = & 11 & \\ \frac{-3x}{-3} & = & \frac{11}{-3} & \textit{Divide both sides by -3.} \\ x & = & -\frac{11}{3} & \end{array}$$

The solution set is $\{-\frac{11}{3}\}$

Example:
Solve $\frac{x}{3} + \frac{1}{2} = \frac{2}{3}$.

$$\begin{array}{rcll}
\frac{x}{3}+\frac{1}{2} & = & \frac{2}{3} & \\
\frac{x}{3}+\frac{1}{2}-\frac{1}{2} & = & \frac{2}{3}-\frac{1}{2} & \textit{Subtract } \frac{1}{2} \text{ on both sides.} \\
\frac{x}{3} & = & \frac{2}{3}\left(\frac{2}{2}\right)-\frac{1}{2}\left(\frac{3}{3}\right) & \\
\frac{x}{3} & = & \frac{4}{6}-\frac{3}{6} & \\
\frac{x}{3} & = & \frac{1}{6} & \\
3\cdot\frac{x}{3} & = & 3\cdot\frac{1}{6} & \textit{Multiply both sides by 3.} \\
x & = & \frac{1}{2} &
\end{array}$$

The solution set is $\left\{\frac{1}{2}\right\}$.

In order to retain the equality, we must perform the same operation on both sides of the equation. To isolate the variable we want to remember to choose the opposite operation not the opposite number. For example, if we have -5x = 20 then we choose to divide both sides by -5, not 5.

Video Example 01

Multiplying by the Reciprocal

Recall that when multiplying reciprocals the result is 1, for example, $\left(\frac{3}{5}\right)\left(\frac{5}{3}\right)=\frac{15}{15}=1$. We can use this fact when the coefficient of the variable is a fraction.

Example:
Solve $-\frac{4}{5}x-5=15$.

$$\begin{aligned}
-\frac{4}{5}x - 5 &= 15 \\
-\frac{4}{5}x - 5 + 5 &= 15 + 5 && \textit{Add 5 on both sides.} \\
-\frac{4}{5}x &= 20 \\
-\frac{5}{4} \cdot \left(-\frac{4}{5}x\right) &= -\frac{5}{4} \cdot (20) && \textit{Multiply both sides by } -\frac{5}{4}. \\
1x &= -5 \cdot 5 && \textit{Simplify.} \\
x &= -25
\end{aligned}$$

The solution set is $\{-25\}$.

Combining Like Terms and Simplifying

Linear equations typically will not be given in standard form and thus will require some additional preliminary steps. These additional steps are to first simplify the expressions on each side of the equal sign using the order of operations.

Opposite Side Like Terms

Given a linear equation in the form $ax + b = cx + d$ we must combine like terms on opposite sides of the equal sign. To do this we will use the addition or subtraction property of equality to combine like terms on either side of the equation.

Example:
Solve for y: $-2y + 3 = 5y + 17$

$$\begin{array}{rcll}
-2y+3 & = & 5y+17 & \\
-2y+3-5y & = & 5y+17-5y & \textit{Subtract 5y on both sides.} \\
-7y+3 & = & 17 & \\
-7y+3-3 & = & 17-3 & \textit{Subtract 3 on both sides.} \\
-7y & = & 14 & \\
\frac{-7y}{-7} & = & \frac{14}{-7} & \textit{Divide both sides by -7.} \\
y & = & -2 &
\end{array}$$

The solution set is $\{-2\}$.

Homework

Solving in One Step

Exercise:

Problem: Solve for x: $x-5=-8$

Solution:

$x=-3$

Exercise:

Problem: Solve for y: $-4+y=-9$

Exercise:

Problem: Solve for x: $x-\frac{1}{2}=\frac{1}{3}$

Solution:

$x=\frac{5}{6}$

Exercise:

Problem: Solve for x: $x+2\frac{1}{2}=3\frac{1}{3}$

Exercise:

Problem: Solve for x: $4x=-44$

Solution:

$x=-11$

Exercise:

Problem: Solve for a: $-3a=-30$

Exercise:

Problem: Solve for y: $27=9y$

Solution:

$y=3$

Exercise:

Problem: Solve for x: $\frac{x}{3}=-\frac{1}{2}$

Exercise:

Problem: Solve for t: $-\frac{t}{12}=\frac{1}{4}$

Solution:

$t=-3$

Exercise:

Problem: Solve for x: $\frac{2}{7}x=-6$

Solve in Two Steps

Exercise:

Problem: Solve for a: $3a - 7 = 23$

Solution:

$a = 10$

Exercise:

Problem: Solve for y: $-3y + 2 = -13$

Exercise:

Problem: Solve for x: $-4x + 8 = 24$

Solution:

$x = -4$

Exercise:

Problem: Solve for x: $\frac{1}{8}x - \frac{1}{2} = -\frac{3}{4}$

Exercise:

Problem: Solve for y: $3 - 2y = -11$

Solution:

$y = 7$

Exercise:

Problem: Solve for x: $-10 = 2x - 5$

Exercise:

Problem: Solve for a: $4a - \frac{2}{3} = -\frac{1}{6}$

Solution:

$a = \frac{1}{8}$

Exercise:

Problem: Solve for x: $\frac{3}{5}x - \frac{1}{2} = \frac{1}{10}$

Exercise:

Problem: Solve for y: $-\frac{4}{5}y + \frac{1}{3} = \frac{1}{15}$

Solution:

$y = \frac{1}{3}$

Exercise:

Problem: Solve for x: $-x - 5 = -2$

Solve in Multiple Steps

Exercise:

Problem: Solve for x: $3x - 5 = 2x - 17$

Solution:

$x = -12$

Exercise:

Problem: Solve for y: $-2y - 7 = 3y + 13$

Applications of Linear Equations in Health and Science

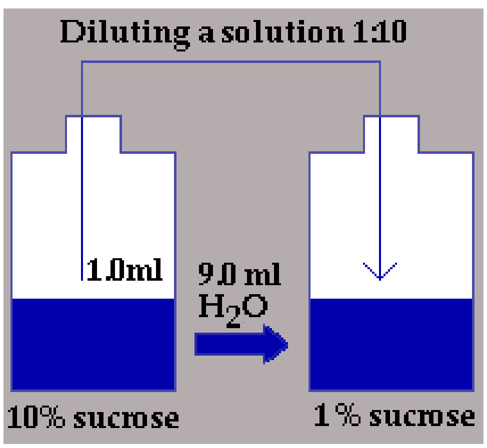
Learning Objectives

By the end of this lesson, you will be able to use linear equations to:

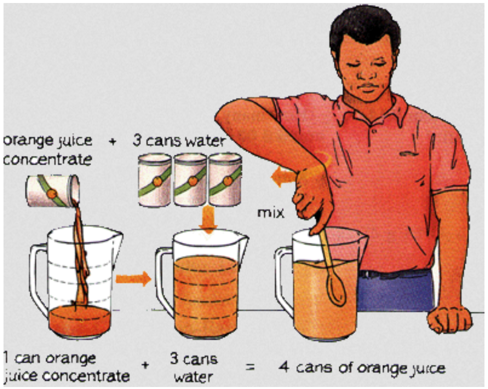
- Solve problems involving the dilution of stock solutions.
- Solve problems using Boyle's Law.

Diluting Stock Solutions

When a solution is ordered, it may be that we can find it already mixed in the correct concentration or percentage. Other times we find a "stock solution" already mixed, but it is too strong. We can use it if we first dilute it so the concentration is decreased.



In this image, water is added to a 10% stock solution to dilute it to 1%. Note that 9.0 mL of water is added to 1.0 mL of the stock solution. This gives a **chemical-to-solution** ratio of 1 to 10 since there is 1 mL of chemical and 1 + 9 = 10 mL of solution.



This image shows the man using a strong stock concentrate (100% pure orange juice) and adding water to dilute it to a weaker strength. Note that the ratio of this solution is 1 to 4. Remember this ratio is “chemical-to-solution” and not chemical-to-water. A 1 to 4 ratio is the same as a 25% solution. So he used a 100% stock solution and diluted it down to a 25% solution.

When we are diluting stock solutions, we will need to determine how much of the strong stock solution is needed, as well as how much water, to create the ordered solution. We will use a formula for the calculation and then we will write a sentence to describe how to prepare the ordered solution.

The formula we will use is: $C_1V_1 = C_2V_2$, where

C_1 is the concentration or strength of the premixed stock solution.
V_1 is the volume of the premixed stock solution that will be used.
C_2 is the concentration or strength of the desired resulting solution.
V_2 is the volume of the desired resulting solution.

As you are reading these examples, notice how the percent sign is used as a label and that the number is not changed to a decimal, even though we are multiplying. (The percentages could be changed to decimals as long as both are changed, but it is not necessary in this situation.) Be sure to answer both

questions - how much stock is needed and how to prepare it.

Example:
Exercise:

Problem:

The order is for 1000 mL of a 10% solution to be made from a 25% stock solution. How much stock solution is needed to prepare this order? Describe how to prepare the ordered solution.

Solution:

The formula to use: $C_1V_1 = C_2V_2$.

Determine what values are known:

$C_1 = 25\%$
$V_1 = \text{unknown}$
$C_2 = 10\%$
$V_2 = 1000 \text{ mL}$

Plugging these values into the formula and then solving the equation, we obtain:
Equation:

$$(25)V_1 = (10)\,(1000)$$
$$\frac{25V_1}{25} = \frac{10000}{25}$$
$$V_1 = 400 \text{ mL}$$

So, 400 mL of stock solution is needed. This is the answer to the first question.

Now for the second question: How do we prepare the solution? Since the total volume of the ordered solution must be 1000 mL, we need to add 600 mL of water to the 400 mL of stock solution.

Example:
Exercise:

Problem:

The order is for 500 mL of a 0.2% solution to be made from a 0.5% stock solution. How much stock solution is needed to prepare this order? Describe how to prepare the ordered solution.

Solution:

The formula to use: $C_1V_1 = C_2V_2$.

Determine what values are known:

$C_1 = 0.5\%$
$V_1 = \text{unknown}$
$C_2 = 0.2\%$
$V_2 = 500 \text{ mL}$

Plugging these values into the formula and then solving the equation, we obtain:
Equation:

$$(0.5)V_1 = (0.2)\,(500)$$
$$\frac{0.5V_1}{0.5} = \frac{100}{0.5}$$
$$V_1 = 200 \text{ mL}$$

So, 200 mL of stock solution is needed. This is the answer to the first question.

Now for the second question: How do we prepare the solution? Since the total volume of the ordered solution must be 500 mL, we need to add 300 mL of water to the 200 mL of stock solution.

Boyle's Law

Boyle's Law relates the pressure and volume of an ideal gas. This law states that, at constant temperature for a fixed mass, the product of absolute pressure and volume is always constant.

The Boyle's Law formula is $P_1V_1 = P_2V_2$, where

P_1 is the initial pressure of a gas.
V_1 is the initial volume.
P_2 is the final pressure.
V_2 is the final volume.

Example:
Exercise:

Problem:

A respiratory therapist is monitoring a patient. The 150 mL of oxygen is at a pressure of 750 mmHg. If the pressure changed to 200 mmHg while the temperature remained the same, what would be the resulting volume?

Solution:

The formula to use: $P_1V_1 = P_2V_2$

Determine what values are known:

$P_1 = 750$ mmHg
$V_1 = 150$ mL
$P_2 = 200$ mmHg
$V_2 =$ unknown

Plugging these values into the formula and then solving the equation, we obtain:
Equation:

$$(750)\,(150) = 200V_2$$
$$\frac{112500}{200} = \frac{200V_2}{200}$$
$$562.5 = V_2$$

So, the resulting volume is 562.5 mL.

Example:
Exercise:

Problem:

A container of 200 mL of oxygen is at a pressure of 625 mmHg. If the pressure changed to 350 mmHg, what would be the resulting volume?

Solution:

The formula to use: $P_1V_1 = P_2V_2$

Determine what values are known:

$P_1 = 625 \text{ mmHg}$
$V_1 = 200 \text{ mL}$
$P_2 = 350 \text{ mmHg}$
$V_2 = \text{unknown}$

Plugging these values into the formula and then solving the equation, we obtain:
Equation:

$$(625)(200) = 350V_2$$
$$\frac{125000}{350} = \frac{350V_2}{350}$$
$$357 \approx V_2$$

So, the resulting volume is 357 mL.

Homework

For each of the following exercises, use the formula $C_1V_1 = C_2V_2$ to determine how much stock solution must be used to prepare the ordered solution and describe how to prepare the solution.
Exercise:

Problem:

Prepare 50 mL of a 0.05% solution using a 0.2% stock solution.

Solution:

Use 12.5 mL of the stock solution and add 37.5 mL of water to make 50 mL.

Exercise:

Problem:

Prepare 200 mL of a 20% solution using a 40% stock solution.

Exercise:

Problem:

Prepare 200 mL of a 0.1% solution using a 0.45% stock solution.

Solution:

Use 44.4 mL of the stock solution and add 155.6 mL of water to make 200 mL.

Exercise:

Problem:

Prepare 500 mL of a 25% solution using a 50% stock solution.

Exercise:

Problem:

Prepare 100 mL of a 0.25% solution using a 0.3% stock solution.

Solution:

Use 83.3 mL of stock solution and add 16.7 mL of water to make 100 mL.

Exercise:

Problem:

Prepare 400 mL of a 3% solution using a 15% stock solution.

Exercise:

Problem:

Prepare 500 mL of a 10% bleach solution for cleaning made with 100% bleach.

Solution:

Use 50 mL of pure bleach stock solution and add 450 mL of water to make 500 mL. This is a 10% bleach solution used for cleaning.

For the following exercises, use the formula for Boyle's Law, $P_1V_1 = P_2V_2$, to answer the question.

Exercise:

Problem:

A respiratory therapist is monitoring a patient. The 175 mL of oxygen is at a pressure of 650 mmHg. While keeping the same temperature, what is the resulting volume if the pressure changed to 200 mmHg?

Exercise:

Problem:

A respiratory therapist is monitoring a patient. The 175 mL of oxygen is at a pressure of 650 mmHg. While keeping the same temperature, what is the resulting volume if the pressure changed to 375 mmHg?

Solution:

303.33 mL

Exercise:

Problem:

A respiratory therapist is monitoring a patient. The 175 mL of oxygen is at a pressure of 650 mmHg. While keeping the same temperature, what is the resulting volume if the pressure changed to 100 mmHg?

Intro to Proportions

Learning Objectives

By the end of this section, you will be able to:

- Identify a proportion
- Use cross products to determine whether a given proportion is true or not.

Use the Definition of Proportion

In the section on Ratios and Rates we saw some ways they are used in our daily lives. When two ratios or rates are stated to be equal, the equation relating them is called a **proportion**.

Note:
Proportion
A proportion is an equation of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0, d \neq 0$.
The proportion states two ratios or rates are equal. The proportion is read "a is to b, as c is to d".

The equation $\frac{1}{2} = \frac{4}{8}$ is a proportion because the two fractions are equal. The proportion $\frac{1}{2} = \frac{4}{8}$ is read "1 is to 2 as 4 is to 8".

If we compare quantities with units, we have to be sure we are comparing them in the right order. For example, in the proportion $\frac{20\text{ students}}{1\text{ teacher}} = \frac{60\text{ students}}{3\text{ teachers}}$ we compare the number of students to the number of teachers. We put students in the numerators and teachers in the denominators.

Example:
Exercise:

Problem: Write each sentence as a proportion:

3 is to 7 as 15 is to 35.
5 hits in 8 at bats is the same as 30 hits in 48 at-bats.
$\$1.50$ for 6 ounces is equivalent to $\$2.25$ for 9 ounces.

Solution:

a	
	3 is to 7 as 15 is to 35.
Write as a proportion.	$\frac{3}{7}=\frac{15}{35}$

b	
	5 hits in 8 at-bats is the same as 30 hits in 48 at-bats.
Write each fraction to compare hits to at-bats.	$\frac{\text{hits}}{\text{at-bats}}=\frac{\text{hits}}{\text{at-bats}}$
Write as a proportion.	

	$\frac{5}{8} = \frac{30}{48}$

c	
	\$1.50 for 6 ounces is equivalent to \$2.25 for 9 ounces.
Write each fraction to compare dollars to ounces.	$\frac{\$}{\text{ounces}} = \frac{\$}{\text{ounces}}$
Write as a proportion.	$\frac{1.50}{6} = \frac{2.25}{9}$

Example:
Exercise:

Problem: Write each sentence as a proportion:

5 is to 9 as 20 is to 36.
7 hits in 11 at-bats is the same as 28 hits in 44 at-bats.
\$2.50 for 8 ounces is equivalent to \$3.75 for 12 ounces.

Solution:

$\frac{5}{9} = \frac{20}{36}$
$\frac{7}{11} = \frac{28}{44}$

$\frac{2.50}{8} = \frac{3.75}{12}$

Example:
Exercise:

Problem: Write each sentence as a proportion:

6 is to 7 as 36 is to 42.
8 adults for 36 children is the same as 12 adults for 54 children.
$\$3.75$ for 6 ounces is equivalent to $\$2.50$ for 4 ounces.

Solution:

$\frac{6}{7} = \frac{36}{42}$
$\frac{8}{36} = \frac{12}{54}$
$\frac{3.75}{6} = \frac{2.50}{4}$

Look at the proportions $\frac{1}{2} = \frac{4}{8}$ and $\frac{2}{3} = \frac{6}{9}$. From our work with equivalent fractions we know these equations are true. But how do we know if a proportion is true with equivalent fractions if it contains fractions with larger numbers?

To determine if a proportion is true, we find the **cross products** of each proportion. To find the cross products, we multiply each denominator with the opposite numerator (diagonally across the equal sign). The results are called a cross products because of the cross formed. The cross products of a proportion must be equal for the proportion to be true.

$8 \cdot 1 = 8 \quad 2 \cdot 4 = 8$

$$\frac{1}{2} = \frac{4}{8}$$

$9 \cdot 2 = 18 \quad 3 \cdot 6 = 18$

$$\frac{2}{3} = \frac{6}{9}$$

Note:
Cross Products of a Proportion
For any true proportion of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0, d \neq 0$, its cross products are equal.

$a \cdot d = b \cdot c$

$$\frac{a}{b} = \frac{c}{d}$$

Cross products can be used to test whether a proportion is true. To test whether an equation makes a true proportion, we find the cross products. If they are the equal, we have a true proportion.

Example:
Exercise:

Problem: Determine whether each equation is a true proportion:

$$\frac{4}{9} = \frac{12}{28}$$
$$\frac{17.5}{37.5} = \frac{7}{15}$$

Solution:

To determine if the equation is a true proportion, we find the cross products. If they are equal, the equation is a true proportion.

a	
	$\frac{4}{9} = \frac{12}{28}$
Find the cross products.	4(28) ≠ 9(12)

Since the cross products are not equal, $4 \cdot 28 \neq 9 \cdot 12$, the equation is not a true proportion.

b	
	$\frac{17.5}{37.5} = \frac{7}{15}$
Find the cross products.	15(17.5) = 37.5(7)

Since the cross products are equal, $15 \cdot 17.5 = 37.5 \cdot 7$, the equation is a true proportion.

Example:
Exercise:

Problem: Determine whether each equation is a true proportion:

$\frac{7}{9} = \frac{54}{72}$
$\frac{24.5}{45.5} = \frac{7}{13}$

Solution:

no, 7(72) ≠ 9(54)
yes, 7(45.5)= 13(24.5)

Example:
Exercise:

Problem: Determine whether each equation is a true proportion:

$\frac{8}{9} = \frac{56}{73}$
$\frac{28.5}{52.5} = \frac{8}{15}$

Solution:

no, 8(73) ≠ 9(56)
no, 8(52.5) ≠ 15(28.5)

Homework

In the following exercises, write each sentence as a proportion.
Exercise:

Problem: 4 is to 15 as 36 is to 135.

Solution:

$\frac{4}{15} = \frac{36}{135}$

Exercise:

Problem: 7 is to 9 as 35 is to 45.

Exercise:

Problem: 12 is to 5 as 96 is to 40.

Solution:

$\frac{12}{5} = \frac{96}{40}$

Exercise:

Problem: 15 is to 8 as 75 is to 40.

Exercise:

Problem: 5 wins in 7 games is the same as 115 wins in 161 games.

Solution:

$\frac{5}{7} = \frac{115}{161}$

Exercise:

Problem: 4 wins in 9 games is the same as 36 wins in 81 games.

Exercise:

Problem:

8 campers to 1 counselor is the same as 48 campers to 6 counselors.

Solution:

$\frac{8}{1} = \frac{48}{6}$

Exercise:

Problem:

6 campers to 1 counselor is the same as 48 campers to 8 counselors.

Exercise:

Problem: $9.36 for 18 ounces is the same as $2.60 for 5 ounces.

Solution:

$\frac{9.36}{18} = \frac{2.60}{5}$

Exercise:

Problem: $3.92 for 8 ounces is the same as $1.47 for 3 ounces.

Exercise:

Problem: $18.04 for 11 pounds is the same as $4.92 for 3 pounds.

Solution:

$\frac{18.04}{11} = \frac{4.92}{3}$

Exercise:

Problem: $12.42 for 27 pounds is the same as $5.52 for 12 pounds.

In the following exercises, determine whether each equation is a true proportion.

Exercise:

Problem: $\frac{7}{15} = \frac{56}{120}$

Solution:

yes

Exercise:

Problem: $\frac{5}{12} = \frac{45}{108}$

Exercise:

Problem: $\frac{11}{6} = \frac{21}{16}$

Solution:

no

Exercise:

Problem: $\frac{9}{4} = \frac{39}{34}$

Exercise:

Problem: $\frac{12}{18} = \frac{4.99}{7.56}$

Solution:

no

Exercise:

Problem: $\frac{9}{16} = \frac{2.16}{3.89}$

Exercise:

Problem: $\frac{13.5}{8.5} = \frac{31.05}{19.55}$

Solution:

yes

Exercise:

Problem: $\frac{10.1}{8.4} = \frac{3.03}{2.52}$

Applications of Proportions

Learning Objectives

By the end of this section, you will be able to:

- Solve proportions.
- Solve general application problems using proportions.
- Solve problems involving Charles Law.

Solve Proportions

When solving a proportion for a variable, we'll use the fact that the cross products of a proportion are equal.

We can find the cross products of the proportion and then set them equal. Then we solve the resulting equation using our familiar techniques.

Example:
Exercise:

Problem: Solve: $\frac{144}{a} = \frac{9}{4}$.

Solution:

Notice that the variable is in the denominator, so we will solve by finding the cross products and setting them equal.

	$\frac{144}{a} = \frac{9}{4}$
Find the cross products and set them equal.	$4 \cdot 144 = a \cdot 9$
Simplify.	$576 = 9a$
Divide both sides by 9.	$\frac{576}{9} = \frac{9a}{9}$
Simplify.	$64 = a$

Check your answer.

	$\frac{144}{a} = \frac{9}{4}$
Substitute $a = 64$	$\frac{144}{64} \stackrel{?}{=} \frac{9}{4}$
Show common factors..	

	$\frac{9 \cdot 16}{4 \cdot 16} \stackrel{?}{=} \frac{9}{4}$
Simplify.	$\frac{9}{4} = \frac{9}{4}$ ✓

Note:
Try It
Exercise:

Problem: Solve the proportion: $\frac{91}{b} = \frac{7}{5}$.

Solution:

65

Example:
Exercise:

Problem: Solve: $\frac{52}{91} = \frac{-4}{y}$.

Solution:

Find the cross products and set them equal.	$\frac{52}{91} = \frac{-4}{y}$
	$y \cdot 52 = 91(-4)$
Simplify.	$52y = -364$
Divide both sides by 52.	$\frac{52y}{52} = \frac{-364}{52}$
Simplify.	$y = -7$

Check:	
	$\frac{52}{91} = \frac{-4}{y}$
Substitute $y = -7$	$\frac{52}{91} \stackrel{?}{=} \frac{-4}{-7}$
Show common factors.	

	$\frac{13 \cdot 4}{13 \cdot 4} \stackrel{?}{=} \frac{-4}{-7}$
Simplify.	$\frac{4}{7} = \frac{4}{7}$ ✓

Note:
Try It
Exercise:

Problem: Solve the proportion: $\frac{84}{98} = \frac{-6}{x}$.

Solution:

-7

Solve Applications Using Proportions

The strategy for solving applications that we have used earlier in this chapter, also works for proportions, since proportions are equations. When we set up the proportion, we must make sure the units are correct—the units in the numerators match and the units in the denominators match.

Example:
Exercise:

Problem:

When pediatricians prescribe acetaminophen to children, they prescribe 5 milliliters (mL) of acetaminophen for every 25 pounds of the child's weight. If Zoe weighs 80 pounds, how many milliliters of acetaminophen will her doctor prescribe?

Solution:

Identify what you are asked to find.	How many mL of acetaminophen the doctor will prescribe.
Choose a variable to represent it.	Let a = mL of acetaminophen.
Write a sentence that gives the information to find it.	If 5 ML is prescribed for every 25 pounds, how much will be prescribed for 80 pounds?
Translate into a proportion.	$\frac{\text{mL}}{\text{pounds}} = \frac{\text{mL}}{\text{pounds}}$
Substitute the given values – be careful of the units.	$\frac{5}{25} = \frac{a}{80}$
Find the cross products and set them equal.	$25a = 400$
Divide both sides by 25 to isolate the variable.	$\frac{25a}{25} = \frac{400}{25}$
Simplify.	$a = 16$
Check if the answer is reasonable.	Yes, since 80 is about 3 times 25, the medicine should be about 3 times 5.
Write a complete sentence.	The pediatrician would prescribe 16 mL of acetaminophen to Zoe.

Note:
Try It
Exercise:

Problem:

Pediatricians prescribe 5 milliliters (ml) of acetaminophen for every 25 pounds of a child’s weight. How many milliliters of acetaminophen will the doctor prescribe for Emilia, who weighs 60 pounds?

Solution:

12 ml

Note:
Try It
Exercise:

Problem:

For every 1 kilogram (kg) of a child’s weight, pediatricians prescribe 15 milligrams (mg) of a fever reducer. If Isabella weighs 12 kg, how many milligrams of the fever reducer will the pediatrician prescribe?

Solution:

180 mg

Example:
Exercise:

Problem:

One brand of microwave popcorn has 120 calories per serving. A whole bag of this popcorn has 3.5 servings. How many calories are in a whole bag of this microwave popcorn?

Solution:

Identify what you are asked to find.	How many calories are in a whole bag of microwave popcorn?
Choose a variable to represent it.	Let c = number of calories.
Write a sentence that gives the information to find it.	If there are 120 calories per serving, how many calories are in a whole bag with 3.5 servings?
Translate into a proportion.	$\frac{\text{calories}}{\text{serving}} = \frac{\text{calories}}{\text{serving}}$
Substitute the given values – be careful of the units.	$\frac{120}{1} = \frac{c}{3.5}$
Find the cross products and set them equal.	$c = 120(3.5)$
Simplify.	$c = 420$
Check if the answer is reasonable.	Yes, since 3.5 is between 3 and 4, the total calories should between 360 (3 · 120) and 480 (4 · 120).
Write a complete sentence.	The whole bag of microwave popcorn has 420 calories.

Note:
Try It
Exercise:

Problem:

Marissa loves the Caramel Macchiato at the coffee shop. The 16 oz. medium size has 240 calories. How many calories will she get if she drinks the large 20 oz. size?

Solution:

300

Charles Law

Charles Law is an experimental gas law that describes how gases tend to expand when heated. According to this law, if the pressure of the gas is held constant, then the relationship between the volume and temperature of the gas can be modeled by the formula

Equation:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Note that this formula is a proportion. If we know the initial volume and temperature of a gas, then we can find the resulting volume from changing the temperature or vise versa, by plugging in the three known values and solving the proportion to find the unknown value.

Example:

Exercise:

Problem:

If 425 mL of nitrogen, originally at a temperature of 293 K, were rapidly squeezed so the resulting volume was 350 mL, what would the temperature change to?

Solution:

Plugging in the known values into the formula and solving the proportion for the resulting temperature, we obtain:

Equation:

$$\frac{425 \text{ mL}}{293 \text{ K}} = \frac{350 \text{ mL}}{T_2}$$
$$425T_2 = (293)(350)$$
$$\frac{425T_2}{425} = \frac{102550}{425}$$
$$T_2 \approx 241 \text{ K}$$

Homework

Solve Proportions

In the following exercises, solve each proportion.

Exercise:

Problem: $\frac{x}{56} = \frac{7}{8}$

Solution:

49

Exercise:

Problem: $\frac{n}{91} = \frac{8}{13}$

Exercise:

Problem: $\frac{49}{63} = \frac{z}{9}$

Solution:

7

Exercise:

Problem: $\frac{56}{72} = \frac{y}{9}$

Exercise:

Problem: $\frac{5}{a} = \frac{65}{117}$

Solution:

9

Exercise:

Problem: $\frac{4}{b} = \frac{64}{144}$

Exercise:

Problem: $\frac{98}{154} = \frac{-7}{p}$

Solution:

-11

Exercise:

Problem: $\frac{72}{156} = \frac{-6}{q}$

Exercise:

Problem: $\frac{a}{-8} = \frac{-42}{48}$

Solution:

7

Exercise:

Problem: $\frac{b}{7} = \frac{30}{42}$

Exercise:

Problem: $\frac{2.6}{3.9} = \frac{c}{3}$

Solution:

2

Exercise:

Problem: $\frac{2.7}{3.6} = \frac{d}{4}$

Exercise:

Problem: $\frac{2.7}{j} = \frac{0.9}{0.2}$

Solution:

0.6

Exercise:

Problem: $\frac{2.8}{k} = \frac{2.1}{1.5}$

Exercise:

Problem: $\frac{\frac{1}{2}}{1} = \frac{m}{8}$

Solution:

4

Exercise:

Problem: $\frac{\frac{1}{3}}{3} = \frac{9}{n}$

Solve Applications Using Proportions

In the following exercises, set up and solve a proportion to solve the problem.

Exercise:

Problem:

Pediatricians prescribe 5 milliliters (mL) of acetaminophen for every 25 pounds of a child’s weight. How many milliliters of acetaminophen will the doctor prescribe for Jocelyn, who weighs 45 pounds?

Solution:

9 mL

Exercise:

Problem:

Brianna, who weighs 6 kg, just received her shots and needs a pain killer. The pain killer is prescribed for children at 15 milligrams (mg) for every 1 kilogram (kg) of the child’s weight. How many milligrams will the doctor prescribe?

Exercise:

Problem:

At the gym, Carol takes her pulse for 10 sec and counts 19 beats. How many beats per minute is this? Has Carol met her target heart rate of 140 beats per minute?

Solution:

114, no

Exercise:

Problem:

Kevin wants to keep his heart rate at 160 beats per minute while training. During his workout he counts 27 beats in 10 seconds. How many beats per minute is this? Has Kevin met his target heart rate?

Exercise:

Problem:

A new energy drink advertises 106 calories for 8 ounces. How many calories are in 12 ounces of the drink?

Solution:

159 cal

Exercise:

Problem:

One 12 ounce can of soda has 150 calories. If Josiah drinks the big 32 ounce size from the local mini-mart, how many calories does he get?

Exercise:

Problem:

Karen eats $\frac{1}{2}$ cup of oatmeal that counts for 2 points on her weight loss program. Her husband, Joe, can have 3 points of oatmeal for breakfast. How much oatmeal can he have?

Solution:

$\frac{3}{4}$ cup

Exercise:

Problem:

An oatmeal cookie recipe calls for $\frac{1}{2}$ cup of butter to make 4 dozen cookies. Hilda needs to make 10 dozen cookies for the bake sale. How many cups of butter will she need?

Exercise:

Problem:

Mixing a concentrate Sam bought a large bottle of concentrated cleaning solution at the warehouse store. He must mix the concentrate with water to make a solution for washing his windows. The directions tell him to mix 3 ounces of concentrate with 5 ounces of water. If he puts 12 ounces of concentrate in a bucket, how many ounces of water should he add? How many ounces of the solution will he have altogether?

Solution:

20 oz., 32 oz.

Exercise:

Problem:

Mixing a concentrate Travis is going to wash his car. The directions on the bottle of car wash concentrate say to mix 2 ounces of concentrate with 15 ounces of water. If Travis puts 6 ounces of concentrate in a bucket, how much water must he mix with the concentrate?

Exercise:

Use Charles Law to fill in the table.

Problem:

Volume	Temperature
Initial $V_1 = 500$ mL	Initial $T_1 = 310$ K
Resulting $V = 400$ mL	Resulting $T =$
Resulting $V = 150$ mL	Resulting $T =$

Solution:

Volume	Temperature
Initial V_1 = 500 mL	Initial T_1 = 310 K
Resulting V = 400 mL	Resulting T = 248 K
Resulting V = 150 mL	Resulting T = 93 K

Exercise:

Use Charles Law to fill in the table.

Problem:

Volume	Temperature
Initial V_1 = 425 mL	Initial T_1 = 293 K
Resulting V = 300 mL	Resulting T =
Resulting V = 250 mL	Resulting T =

Variation

Learning Objectives

By the end of this lesson, you will be able to:

- Solve and identify direct variation problems.
- Solve and identify inverse variation problems.
- Find the constant of proportionality.

Direct Variation

A relationship in which one quantity is a constant multiplied by another quantity is called **direct variation**. Each variable in this type of relationship varies directly with the other.

The constant multiplier is called the **constant of proportionality** and is represented by the variable k.

One characteristic of direct variation between two variables is that if one variable increases, the other variable increases as well.

Example:
When the volume of a gas is kept constant, the relationship between the pressure and temperature of the gas is an example of direct variation. In the table below, a gas is kept at a constant volume while the pressure is increased. Then, the resulting temperature is also recorded.

Pressure	Temperature	Constant of Proportionality k
Initial $P_1 = 750$ mmHg	Initial $T_1 = 300$ K	
$P_2 = 812.5$ mmHg	$T_2 = 325$ K	
$P_3 = 1000$ mmHg	$T_3 = 400$ K	

Notice how as the pressure increases, so does the temperature. This is a characteristic of direct variation. Another characteristic of direct variation is that one quantity must be a constant (k) multiplied by the other quantity. To find the constant of proportionality, k, we divide each pressure by its respective temperature.

Equation:

$$\frac{750}{300} = 2.5; \quad \frac{812.5}{325} = 2.5; \quad \frac{1000}{400} = 2.5$$

Thus, the constant of proportionality is k = 2.5 mmHg/K. Note that the we determine the units mmHg/K by noticing that this value of k was calculated by dividing each pressure, which is measured in mmHg, by the respective temperature, which is measured in degrees Kelvin (K).

This value is a constant of proportionality because you can multiply each temperature by this value and obtain the respective pressure. For example, $300(2.5) = 750$.
There is another constant of proportionality possible, if we instead divide each temperature by the respective pressure.
Equation:

$$\frac{300}{750} = 0.4; \quad \frac{325}{812.5} = 0.4; \quad \frac{400}{1000} = 0.4$$

Thus, another constant of proportionality for this relationship is $k = 0.4$ K/mmHg. You can verify that this is another constant of proportionality by multiplying this value by each pressure to see if you obtain the respective temperature.

Inverse Variation

Another type of variation that we will look at is **inverse variation**. When the product of two variables is constant, in other words when their product

is always equal to the same value, the relationship between the two variables is called inverse variation. The product that stays constant is called the constant of proportionality and is represented with the letter k. (Note that since multiplication is commutative, which means the order doesn't matter, there is only one constant of proportionality possible with inverse variation.)

A characteristic of inverse variation between two variables is that as one variable increases, the other variable decreases, and vice versa.

Example:
Exercise:

Problem:

Recall that, according to Boyle's Law, if the temperature of a gas is kept constant, then the relationship between the pressure and volume of the gas can be modeled by the formula $P_1V_1 = P_2V_2$.

This formula was used to determine the volume of a gas, under various amounts of pressure. The results are recorded in the table below.

Volume	Pressure	Constant of Proportionality, k
Initial V_1 = 175 mL	Initial P_1 = 650 mmHg	
Resulting V = 568.75 mL	Resulting P = 200 mmHg	
Resulting $V \approx$ 303.33 mL	Resulting P = 375 mmHg	
Resulting V = 1137.5 mL	Resulting P = 100 mmHg	

Notice how as the pressure increases, the volume decreases. This is a characteristic of inverse variation. Another characteristic of inverse variation is that the product of the two quantities must be constant. That common product is the constant of proportionality, k. To find the constant of proportionality, we multiply each pressure and it's respective volume. We should get the same result each time.

The following table shows the products for each pair. Note that the volumes were found using the Boyle's Law formula and the third volume was rounded, which is why that product is slightly off. However, if you use the formula to find the exact volume (keeping it as a fraction) and used that value for the volume instead of the rounded value, you will find that the resulting product will be exactly the same as the rest.

Volume	Pressure	Constant of Proportionality, k
Initial $V_1 = 175$ mL	Initial $P_1 = 650$ mmHg	113750
Resulting $V = 568.75$ mL	Resulting $P = 200$ mmHg	113750
Resulting $V \approx 303.33$ mL	Resulting $P = 375$ mmHg	113748.75 (off due to rounding)
Resulting $V = 1137.5$ mL	Resulting $P = 100$ mmHg	113750

Therefore, the constant of proportionality is $k = 113{,}750$.

Example:

When the concentration of a solution must stay the same, this is an example of variation. In this example, we will determine whether this is direct or inverse variation and find the constant of proportionality.

Exercise:

Problem:

a. How much chemical should be added to obtain a total of 45 mL of a 3:10 solution?
b. How much chemical should be added to obtain a total of 100 mL of a 3:10 solution?
c. How much solution will be needed with 5 mL of chemical, to keep the 3:10 concentration?
d. What type of variation is shown in this data?
e. What is the constant of proportionality?

Solution:

a. Let x = the amount of chemical to be added. Then, we set up the following proportion:

$$\frac{3}{10} = \frac{x}{45}$$

Solving the proportion, we find that we need 13.5 mL of chemical.

b. Let x = the amount of chemical to be added. Then, we set up the following proportion:

$$\frac{3}{10} = \frac{x}{100}$$

Solving the proportion, we find that we need 30 mL of chemical.

c. Let x = the amount of chemical to be added. Then, we set up the following proportion:

$$\frac{3}{10} = \frac{5}{x}$$

Solving the proportion, we find that we need about 16.7 mL of solution (16 and 2/3 mL to be exact).

d. Notice that as the amount of solution increases, so does the amount of chemical. So this is an example of direct variation.

e. To find the constant of proportionality, we can divide the amount of chemical by the amount of solution. This gives us, k = 0.3. We can also divide the amount of solution by the amount of chemical to get another constant of proportionality of k is approximately 3.3.

Homework

For the following exercises, determine whether it is an example of direct or inverse variation and find the constant of proportionality.

Exercise:

Problem:

Gas is purchased at a gas station by 4 separate customers and the number of gallons and total cost are recorded in the table below:

Gallons of gas purchased at a gas station	Total cost of gas purchased at the gas station	Constant of Proportionality, *k*
10 gallons	$21.29	
12.5 gallons	$26.61	
15.75 gallons	$33.53	
24 gallons	$51.10	

Solution:

This is direct variation and k is approximately 0.47 gal/$ or $2.13/gal.

Exercise:

Problem:

A distance of 100 miles is traveled by 4 separate vehicles, traveling at different speeds, and the speed and time to travel for each is recorded below:

Speed traveling 100 miles.	Time to travel 100 miles	Constant of Proportionality, *k*
30 miles per hour	about 3.33 hours	
45 miles per hour	about 2.22 hours	
65 miles per hour	about 1.54 hours	
75 miles per hour	about 1.33 hours	

Exercise:

Problem:

A group of people want to buy a gift for someone that costs $40. Fill in the table of data for number of people and how much they have to contribute. Also find how many people contributed if each person contributed $8.

Number of people	Money to contribute	Constant of proportionality k
1 person	$40	
2 people		
3 people		
	$8	

Solution:

This is inverse variation and $k = 40$.

Number of people	Money to contribute	Constant of Proportionality k
1 person	$40	40
2 people	$20	40
3 people	$13.33	39.99 (off due to rounding)
5 people	$8	40

Exercise:

Problem:

When a force of 12 pounds is used to stretch a spring, it can be stretched to 9 inches. The same spring can be stretched to 30 inches by applying 40 pounds of force. What type of variation is this? What is the constant of proportionality?

Exercise:

Problem:

Use the formula $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ to find the resulting temperature when the pressure of the gas is decreased to 697 mmHg. Then, determine the constant of proportionality and which type of variation this is.

Pressure	Temperature	Constant of Proportionality
Initial $P_1 = 850$ mmHg	Initial $T_1 = 305$K	
$P_2 = 697$ mmHg	Resulting $T_2 = ?$	

Solution:

This is direct variation.

Pressure	Temperature	Constant of Proportionality
Initial $P_1 = 850$ mmHg	Initial $T_1 = 305$K	≈ 2.79
$P_2 = 697$ mmHg	Resulting $T_2 = 250$ K	≈ 2.79

Exercise:

Problem:

If 425 mL of nitrogen, originally at a temperature of 293 K, were rapidly squeezed so the resulting volume was 350 mL, what would the temperature change to? Use Charles Gas Law to answer the question: $\frac{V_1}{T_1} = \frac{V_2}{T_2}$. Then, determine which type of variation this is and find the constant of proportionality.

Exercise:

Problem:

It takes 3 people 40 hours to dig a ditch. The same size ditch can be dug by 5 people in 24 hours. What type of variation is this? What is the constant of proportionality?

Solution:

Inverse variation; $k = 120$

Exercise:

Problem:

Last week you worked 32 hours and earned \$408. Two weeks ago, you earned \$446.25 for 35 hours of work. What type of variation is this? What is the constant of proportionality?

Graphing Solutions

Learning Objectives

By the end of this lesson, you will be able to:

- Graph ordered pairs on a rectangular coordinate system.
- Calculate the slope of a line.
- Interpret the slope, in terms of the input and output.

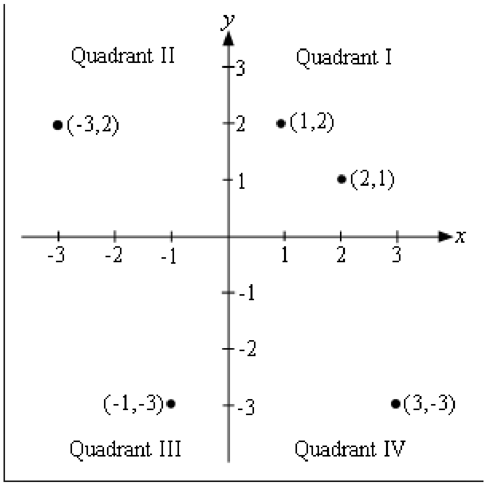
The Rectangular Coordinate System

Grids are used to show the relationship of the result of a formula, equation, or ratio. Given a value for the input, we can find the result or output.

Properties of a **Rectangular Coordinate System**:

- The grid is made of 2 intersecting number lines intersecting at zero.
- The horizontal axis is called the x-axis or the input axis.
- The vertical axis is called the y-axis or the output axis.
- There are four quadrants that are labeled with Roman numerals I, II, III, and IV and this numbering goes counterclockwise.

Notice the use of negative numbers on these number lines. Since we are usually working with measurement numbers we will only use Quadrant I.



There are points plotted and labeled with two numbers. These labels are called **ordered pairs**. The first number is the x-coordinate (or the input value) and the second number is the y-coordinate (or the output value). In this course, since we are only working with measurements, which will always be positive, we will only be working with Quadrant I.

When plotting points on a grid, start at zero. On the x-axis move to the x value and then go up to find the y value. In quadrant I, see the point (2, 1). It is plotted by moving horizontally from zero to 2 and then moving vertically up to 1. Ordered pairs can come from numbers in a table of data, which can be given to us or we can find them by using an equation, formula, or ratio.

The x-coordinates are usually already in the table as they are a sampling of inputs that could work in the relationship.

Each input is substituted into a given formula to find its matching y or output.

After several points are found, they can then be plotted on the graph. The points are connected to make a line or curve. This line is a graphical representation of the relationship in the equation.

Example:
Kool-Aid Solution
Exercise:

Problem:

When mixing Kool-Aid, we are making a solution with a chemical and water. Remember, amount of solution = amount of chemical + amount of water, so amount of Kool-Aid = amount of powder + amount of water.

We are told that it takes 1 tsp of powder to ½ cup of water to make this solution.

Below is a table of data for increasing quantities of solution.

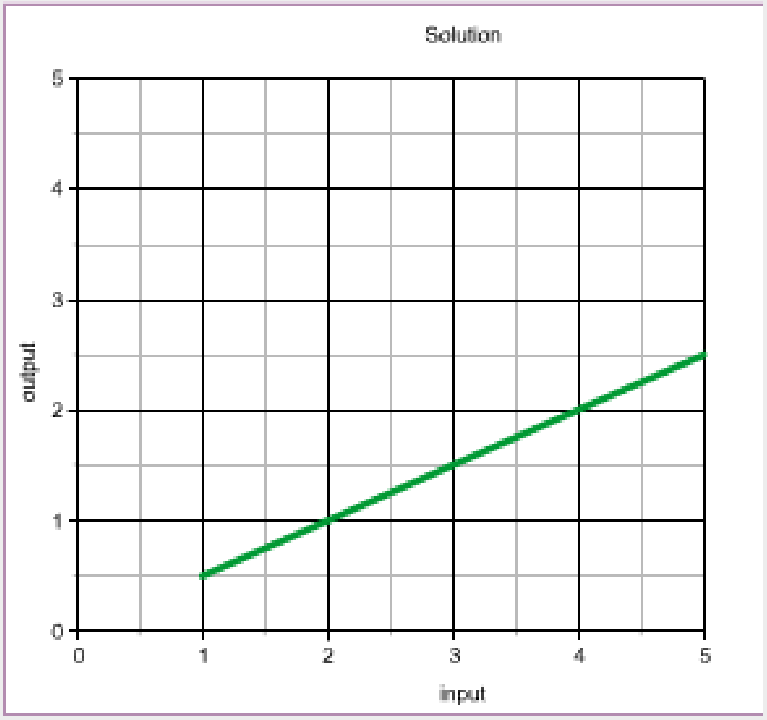
x-axis Chemical input (tsp)	y-axis Solution output (cups)
1	0.5
2	1
3	1.5
4	2
5	2.5

Each of these matching inputs and outputs create an ordered pair. This is a pair that is "ordered" because we list the input first so (1, 0.5) is the first ordered pair while (5, 2.5) is the last. Each of the ordered pairs makes a point on a graph.

Use the ordered pairs from the table to plot points:(1, 0.5), (2, 1), (3, 1.5), (4, 2), (5, 2.5). Using only the first quadrant, plot a point for each of the ordered pairs. Connect the points with a straight line.

Solution:

Plotting points:
For the input, start at zero and go horizontal (right) to reach the input (for example 1) and then go up to reach the output (for example 0.5) and place a dot here.
After several points are plotted they can be connected and we see that they form a line. This line is an estimate of the relationship between input and output.



Calculating and Interpreting Slope

Slope is a numerical description of the steepness of the line on the graph, seen as the "difference of the outputs" divided by the "difference of the inputs."

The formula for slope is: $\boxed{\text{slope} = \frac{\text{output}_1 - \text{output}_2}{\text{input}_1 - \text{input}_2}}$

Slope is a rate so it will have 2 labels as seen from a table of data.

Notice how the formula is using two points with output1 and output2. Notice also that the correlating inputs are in the same order when subtracting.

Example:
Kool-Aid Solution revisited
In the Kool-Aid solution graph, because a line is formed when connecting the points, we can use any two ordered pairs from the table of data to find the slope.
Exercise:

Problem:

Using (2, 1) and (4, 2) we evaluate the formula. Be sure to use outputs in the numerator and inputs in the denominator. Also, reduce the rate to a unit rate.

Solution:

Calculating the slope
Equation:

$$\text{slope} = \frac{1-2}{2-4} = \frac{-1}{-2} = 0.5$$

So, the slope of the line is 0.5. We can verify this by evaluating the formula at two different points, (5, 2.5) and (1, 0.5):
Equation:

$$\text{slope} = \frac{2.5-0.5}{5-1} = \frac{2}{4} = 0.5$$

Interpreting of slope

The slope of the line is 0.5, but what does that mean? Let's write an interpretation of the slope rate with labels, keeping in mind that the outputs are measured in cups and the inputs are measured in teaspoons:

"For every 0.5 cup of solution, there is 1 tsp of chemical."
Or "for every 1 tsp of chemical going in, we get 0.5 cups of solution out."

Notice how the last interpretation uses the words "in" for input and "out" for output. It also uses the correct labels.

Homework

For each of the following problems,

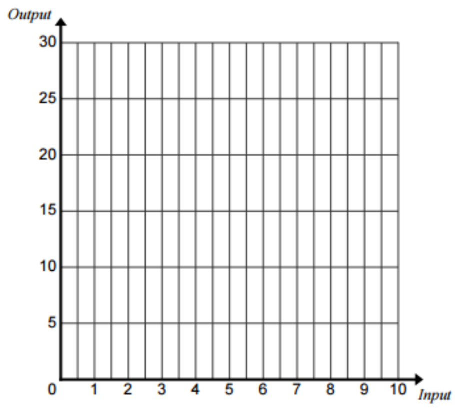
a. Use the given information about solutions and the ratio to complete the table of data.
b. Plot the points on the graph and connect to form a line.
c. Calculate the slope of the line, using the slope formula, and reduce to a unit rate.
d. Write an interpretation for the slope using the correct measuring labels.

Exercise:

Problem:

A 20 mL solution has 7.2 mL of alcohol. Maintain the ratio of 7.2 to 20 to keep the same strength to fill in the table of data.

Input	Output
Alcohol in mL	Solution in mL
	10
5.4	
7.2	20
	25

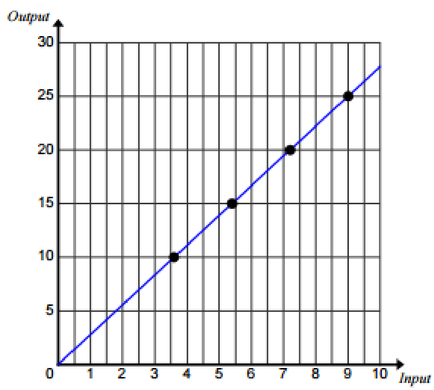


Solution:

a.

Input	Output
Alcohol in mL	Solution in mL
3.6	10
5.4	15
7.2	20
9	25

b.



c. slope is about 2.8.

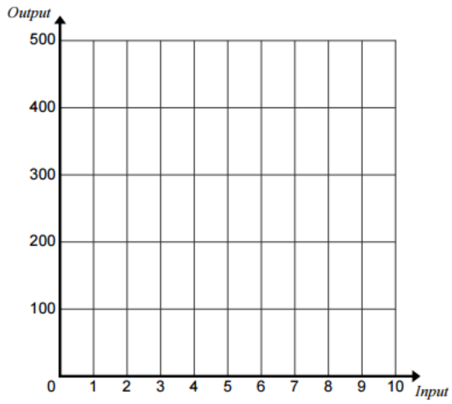
d. For every 2.8 mL of solution there is 1 mL of alcohol.

Exercise:

Problem:

There is a 500 ml solution with 10 grams of salt. Set up proportions to fill in the table of data, maintaining a ratio of 500 to 10.

Input	Output
Salt in grams	Solution in mL
	100
5	
8	
10	500

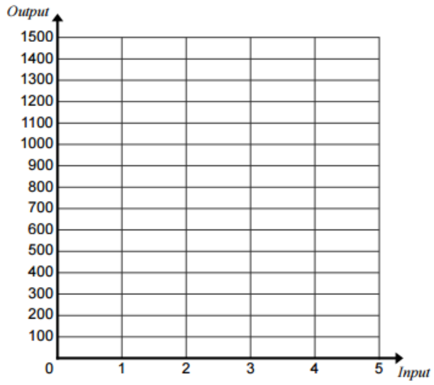


Exercise:

Problem:

There is a 900 fl. oz. solution with 3 oz. of boric acid. Set up proportions to fill in the table of data, maintaining a chemical-to-solution ratio of 3 to 900.

Input	Output
Boric Acid in oz.	Solution in fl. oz.
	300
	600
3	900
4	

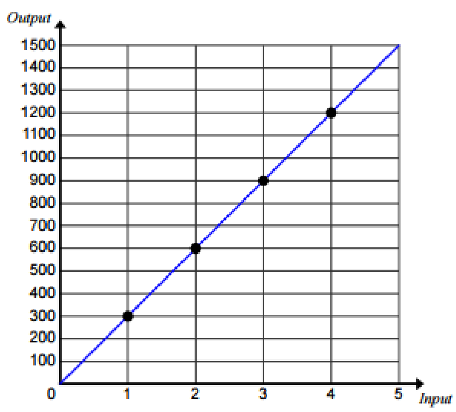


Solution:

a.

Input	Output
Boric Acid in oz.	Solution in fl. oz.
1	300
2	600
3	900
4	1200

b.



c. slope = 300

d. For every 300 fl. oz. of solution, there is 1 of boric acid.

A Review of Percents

Learning Objectives

By the end of this lesson, you will be able to:

- Understand the meaning of percent
- Convert a fraction to a percent
- Convert a percent to a fraction
- Convert a decimal to a percent
- Convert a percent to a decimal

The Meaning of Percent

The word **percent** comes from the Latin word “per centum,” “per” meaning “for each,” and “centum” meaning “hundred.”

Percent (%)
Percent means "per hundred", or “for each hundred”, or “for every hundred.” The symbol % is used to represent the word percent.

Thus, $1\% = \frac{1}{100}$ or $1\% = 0.01.$

Converting A Fraction To A Percent

We can see how a fraction can be converted to a percent by analyzing the method that $\frac{3}{5}$ is converted to a percent. In order to convert $\frac{3}{5}$ to a percent, we need to introduce $\frac{1}{100}$ (since percent means for each hundred).

Example:

$$\begin{aligned}\frac{3}{5} &= \frac{3}{5} \cdot \frac{100}{100} && \text{Multiply the fraction by 1.}\\ &= \frac{3}{5} \cdot 100 \cdot \frac{1}{100} && \text{Since } \frac{100}{100} = 100 \cdot \frac{1}{100}.\\ &= \frac{300}{5} \cdot \frac{1}{100} && \text{Divide 300 by 5.}\\ &= 60 \cdot \frac{1}{100} && \text{Multiply the fractions.}\\ &= 60\% && \text{Replace } \frac{1}{100} \text{ with the } \% \text{ symbol.}\end{aligned}$$

Fraction to Percent

To convert a fraction to a percent, multiply the fraction by 1 in the form $100 \cdot \frac{1}{100}$, then replace $\frac{1}{100}$ with the % symbol.

In the next three examples, we will convert each fraction to a percent.

Example:

$$\begin{aligned}\frac{1}{4} &= \frac{1}{4} \cdot 100 \cdot \frac{1}{100}\\ &= \frac{100}{4} \cdot \frac{1}{100}\\ &= 25 \cdot \frac{1}{100}\\ &= 25\%\end{aligned}$$

Example:

$$\begin{aligned}\frac{8}{5} &= \frac{8}{5} \cdot 100 \cdot \frac{1}{100}\\ &= \frac{800}{5} \cdot \frac{1}{100}\\ &= 160\%\end{aligned}$$

Example:

$$\begin{aligned} \frac{4}{9} &= \frac{4}{9} \cdot 100 \cdot \frac{1}{100} \\ &= \frac{400}{9} \cdot \frac{1}{100} \\ &= (44.4...) \cdot \frac{1}{100} \\ &= \left(44.4\right) \cdot \frac{1}{100} \\ &= 44.4\% \end{aligned}$$

Note:
Try It
Exercise:

Problem: Convert to a percent: $\frac{3}{5}$.

Solution:

37.5%

Converting a Percent to a Fraction

Note:
Convert a percent to a fraction.

Write the percent as a ratio with the denominator 100.
Simplify the fraction if possible.

Example:
Exercise:

Problem: Convert each percent to a fraction:

ⓐ 36%
ⓑ 125%

Solution:
Solution

ⓐ	
	36%
Write as a ratio with denominator 100.	$\frac{36}{100}$
Simplify.	$\frac{9}{25}$

ⓑ	
	125%
Write as a ratio with denominator 100.	$\frac{125}{100}$

Simplify.	$\frac{5}{4}$

Note:
Try It
Exercise:

Problem: Convert each percent to a fraction:

ⓐ 48%
ⓑ 110%

Solution:

ⓐ $\frac{12}{25}$
ⓑ $\frac{11}{10}$

Note:
Try It
Exercise:

Problem: Convert each percent to a fraction:

ⓐ 64%
ⓑ 150%

Solution:

ⓐ $\frac{16}{25}$

ⓑ $\frac{3}{2}$

The previous example shows that a percent can be greater than 1. We saw that 125% means $\frac{125}{100}$, or $\frac{5}{4}$. These are improper fractions, and their values are greater than one.

Example:
Exercise:

Problem: Convert each percent to a fraction:

ⓐ 24.5%
ⓑ $33\frac{1}{3}\%$

Solution:
Solution

ⓐ	
	24.5%
Write as a ratio with denominator 100.	$\frac{24.5}{100}$
Clear the decimal by multiplying numerator and denominator by 10.	$\frac{24.5(10)}{100(10)}$

Multiply.	$\frac{245}{1000}$
Rewrite showing common factors.	$\frac{5 \cdot 49}{5 \cdot 200}$
Simplify.	$\frac{49}{200}$

ⓑ	
	$33\frac{1}{3}\%$
Write as a ratio with denominator 100.	$\frac{33\frac{1}{3}}{100}$
Write the numerator as an improper fraction.	$\frac{\frac{100}{3}}{100}$
Rewrite as fraction division, replacing 100 with $\frac{100}{1}$.	$\frac{100}{3} \div \frac{100}{1}$
Multiply by the reciprocal.	$\frac{100}{3} \cdot \frac{1}{100}$
Simplify.	$\frac{1}{3}$

Note:
Try It
Exercise:

Problem: Convert each percent to a fraction:

ⓐ 64.4%
ⓑ $66\frac{2}{3}\%$

Solution:

ⓐ $\frac{161}{250}$
ⓑ $\frac{2}{3}$

Note:
Try It
Exercise:

Problem: Convert each percent to a fraction:

ⓐ 42.5%
ⓑ $8\frac{3}{4}\%$

Solution:

ⓐ $\frac{113}{250}$
ⓑ $\frac{7}{80}$

Converting a Decimal to a Percent

We can see how a decimal is converted to a percent by analyzing the method that 0.75 is converted to a percent. We need to introduce $\frac{1}{100}$.

$$\begin{aligned} 0.75 &= 0.75 \cdot 100 \cdot \frac{1}{100} && \text{Multiply the decimal by 1.} \\ &= 75 \cdot \frac{1}{100} \\ &= 75\% && \text{Replace } \frac{1}{100} \text{ with the } \% \text{ symbol.} \end{aligned}$$

Decimal to Percent
To convert a fraction to a percent, multiply the decimal by 1 in the form $100 \cdot \frac{1}{100}$, then replace $\frac{1}{100}$ with the % symbol. This amounts to moving the decimal point 2 places to the right.

In the next three examples, we will convert each decimal to a percent.

Example:

$$\begin{aligned} 0.62 &= 0.62 \cdot 100 \cdot \frac{1}{100} \\ &= 62 \cdot \frac{1}{100} \\ &= 62\% \end{aligned}$$

Notice that the decimal point in the original number has been moved to the right 2 places.

Example:

$$\begin{aligned} 8.4 &= 8.4 \cdot 100 \cdot \frac{1}{100} \\ &= 840 \cdot \frac{1}{100} \\ &= 840\% \end{aligned}$$

Notice that the decimal point in the original number has been moved to the right 2 places.

Example:

$$\begin{aligned} 0.47623 &= 0.47623 \cdot 100 \cdot \tfrac{1}{100} \\ &= 0.47623 \cdot \tfrac{1}{100} \\ &= 47.623\% \end{aligned}$$

Notice that the decimal point in the original number has been moved to the right 2 places.

Note:
Try It
Exercise:

Problem: Convert to a percent: 0.0362.

Solution:

3.62%

Converting A Percent To A Decimal

We can see how a percent is converted to a decimal by analyzing the method that 12% is converted to a decimal. We need to introduce $\frac{1}{100}$.

$$\begin{aligned} 12\% &= 12 \cdot \frac{1}{100} && \text{Replace } \% \text{ with } \frac{1}{100}. \\ &= \frac{12}{100} && \text{Multiply the fractions.} \\ &= 0.12 && \text{Divide 12 by 100.} \end{aligned}$$

Percent to Decimal
To convert a percent to a decimal, replace the % symbol with $\frac{1}{100}$, then divide the number by 100. This amounts to moving the decimal point 2 places to the left.

In the next three examples, we will convert each percent to a decimal.

Example:

$$\begin{aligned} 48\% &= 48 \cdot \frac{1}{100} \\ &= \frac{48}{100} \\ &= 0.48 \end{aligned}$$

Notice that the decimal point in the original number has been moved to the left 2 places.

Example:

$$\begin{aligned} 659\% &= 659 \cdot \frac{1}{100} \\ &= \frac{659}{100} \\ &= 6.59 \end{aligned}$$

Notice that the decimal point in the original number has been moved to the left 2 places.

Example:

$$\begin{aligned} 0.4113\% &= 0.4113 \cdot \frac{1}{100} \\ &= \frac{0.4113}{100} \\ &= 0.004113 \end{aligned}$$

Notice that the decimal point in the original number has been moved to the left 2 places.

Note:
Try It
Exercise:

Problem: Convert to a decimal: 4.7%.

Solution:

0.047

Homework

For the following problems, convert each fraction to a percent.
Exercise:

Problem: $\frac{2}{5}$

Solution:

40%

Exercise:

Problem: $\frac{7}{8}$

Exercise:

Problem: $\frac{1}{8}$

Solution:

12.5%

Exercise:

Problem: $\frac{5}{16}$

Exercise:

Problem: $15 \div 22$

Solution:

68.18%

Exercise:

Problem: $\frac{2}{11}$

Exercise:

Problem: $\frac{2}{9}$

Solution:

22.22%

Exercise:

Problem: $\frac{16}{45}$

Exercise:

Problem: $\frac{27}{55}$

Solution:

49.09%

Exercise:

Problem: $\frac{7}{27}$

Exercise:

Problem: 15

Solution:

1500%

Exercise:

Problem: 8

In the following exercises, convert each percent to a fraction and simplify all fractions.

Exercise:

Problem: 4%

Solution:

$\frac{1}{25}$

Exercise:

Problem: 8%

Exercise:

Problem: 17%

Solution:

$\frac{17}{100}$

Exercise:

Problem: 19%

Exercise:

Problem: 52%

Solution:

$\frac{13}{25}$

Exercise:

Problem: 78%

Exercise:

Problem: 125%

Solution:

$\frac{5}{4}$

Exercise:

Problem: 135%

Exercise:

Problem: 37.5%

Solution:

$\frac{3}{8}$

Exercise:

Problem: 42.5%

Exercise:

Problem: 18.4%

Solution:

$\frac{23}{125}$

Exercise:

Problem: 46.4%

Exercise:

Problem: $9\frac{1}{2}\%$

Solution:

$\frac{19}{200}$

Exercise:

Problem: $8\frac{1}{2}\%$

Exercise:

Problem: $5\frac{1}{3}\%$

Solution:

$\frac{4}{75}$

Exercise:

Problem: $6\frac{2}{3}\%$

For the following problems, convert each decimal to a percent.

Exercise:

Problem: 0.36

Solution:

36%

Exercise:

Problem: 0.42

Exercise:

Problem: 0.446

Solution:

44.6%

Exercise:

Problem:0.1298

Exercise:

Problem:4.25

Solution:

425%

Exercise:

Problem:5.875

Exercise:

Problem:86.98

Solution:

8698%

Exercise:

Problem:21.26

Exercise:

Problem:14

Solution:

1400%

Exercise:

Problem:12

For the following problems, convert each percent to a decimal.

Exercise:

Problem:35%

Solution:

0.35

Exercise:

Problem:76%

Exercise:

Problem:18.6%

Solution:

0.186

Exercise:

Problem:67.2%

Exercise:

Problem:9.0145%

Solution:

0.090145

Exercise:

Problem:3.00156%

Exercise:

Problem:0.00005%

Solution:

0.0000005

Exercise:

Problem:0.00034%

Using a Percent Equation to Solve a Percent Problem

Learning Objectives

By the end of this section, you will be able to:

- Translate and solve basic percent equations
- Solve applications of percent
- Find percent increase and percent decrease

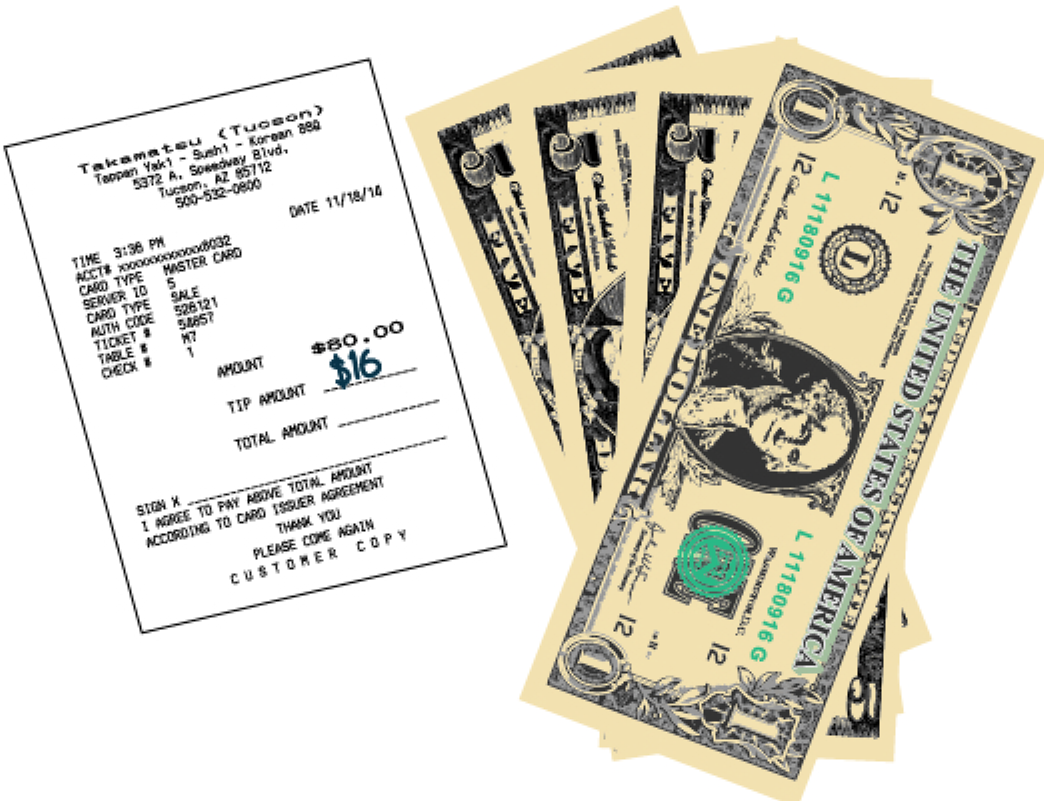
Translate and Solve Basic Percent Equations

We will solve percent equations by using the methods we used to solve equations with fractions or decimals. We will start by translating word sentences into linear equations, and then solve the equations.

We'll look at a common application of percent—tips to a server at a restaurant—to see how to set up a basic percent application.

When Aolani and her friends ate dinner at a restaurant, the bill came to $\$80$. They wanted to leave a 20% tip. What amount would the tip be?

To solve this, we want to find what *amount* is 20% of $\$80$. The $\$80$ is called the *base*. The amount of the tip would be $0.20\,(80)$, or $\$16$ See [link]. To find the amount of the tip, we multiplied the percent by the base.



A 20% tip for an $80 restaurant bill comes out to $16.

Translating Percent Problems into an Equation

To solve a basic percent problem, using an equation, the first step is to translate the problem into an equation. To do this, there are three key words to look for that will need to be translated into a variable or a mathematical symbol.

- “what” translates into a variable since it represents the quantity that needs to be found,
- “of” translates into a multiplication sign (represented by a dot),
- “is” translates into an equal sign.

Keep in mind that the percents in these problems must be changed into decimal form before inserting them into the equation. All other numbers will stay the same.

In the next examples, we will find the amount. We must be sure to change the given percent to a decimal when we translate the words into an equation.

Example:
Exercise:

Problem: What number is 35% of 90?

Solution:

Translate into algebra. Let $n =$ the number.	$\underbrace{\text{What number}}_{n}\ \underbrace{\text{is}}_{=}\ \underbrace{35\%}_{0.35}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{90?}_{90}$ $n = 31.5$
Multiply.	31.5 is 35% of 90

Note:
Try It
Exercise:

Problem: What number is 45% of 80?

Solution:

36

Note:
Try It
Exercise:

Problem: What number is 55% of 60?

Solution:

33

Example:
Exercise:

Problem: 125% of 28 is what number?

Solution:

Translate into algebra. Let $a\ =$ the number.	$\underbrace{\text{125\%}}_{1.25}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{28}}_{28}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{what number?}}_{a}$ $35 = a$
Multiply.	125% of 28 is 35.

Remember that a percent over 100 is a number greater than 1. We found that 125% of 28 is 35, which is greater than 28.

Note:
Try It
Exercise:

Problem: 150% of 78 is what number?

Solution:

117

Note:
Try It
Exercise:

Problem: 175% of 72 is what number?

Solution:

126

In the next examples, we are asked to find the base.

Example:
Exercise:

Problem: Translate and solve: 36 is 75% of what number?

Solution:

Translate. Let $b =$ the number.	$\underbrace{36}_{36}\ \underbrace{\text{is}}_{=}\ \underbrace{75\%}_{0.75}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{what number?}}_{b}$
Divide both sides by 0.75.	$\frac{36}{0.75} = \frac{0.75b}{0.75}$
Simplify.	$48 = b$ 36 is 75% of 48

Note:
Try It
Exercise:

Problem: 17 is 25% of what number?

Solution:

68

Note:
Try It
Exercise:

Problem: 40 is 62.5% of what number?

Solution:

64

Example:
Exercise:

Problem: 6.5% of what number is $\$1.17$?

Solution:

Translate. Let $b =$ the number.	$\underbrace{\text{6.5\%}}_{0.065}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{what number}}_{b}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{\$1.17?}}_{1.17}$
Divide both sides by 0.065.	$\frac{0.065b}{0.065} = \frac{1.17}{0.065}$
Simplify.	$b = 18$ 6.5% of $\$18$ is $\$1.17$.

Note:
Try It
Exercise:

Problem: 7.5% of what number is $\$1.95$?

Solution:

$26

Note:
Try It
Exercise:

Problem: 8.5% of what number is $\$3.06$?

Solution:

$36

In the next examples, we will solve for the percent.

Example:
Exercise:

Problem: What percent of 36 is 9?

Solution:

Translate into algebra. Let $p =$ the percent.	$\underbrace{\text{What percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{36}_{36}\ \underbrace{\text{is}}_{=}\ \underbrace{9?}_{9}$
Divide by 36.	$\frac{36p}{36} = \frac{9}{36}$
Simplify.	$p = \frac{1}{4}$
Convert to decimal form.	$p = 0.25$
Convert to percent.	$p = 25\%$ 25% of 36 is 9.

Note:
Try It
Exercise:

Problem: What percent of 76 is 57?

Solution:

75%

Note:
Try It
Exercise:

Problem: What percent of 120 is 96?

Solution:

80%

Example:
Exercise:

Problem: 144 is what percent of 96?

Solution:

Translate into algebra. Let $p =$ the percent.	$\underbrace{144}_{144}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{what percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{96?}_{96}$
Divide by 96.	$\frac{144}{96} = \frac{96p}{96}$
Simplify.	$1.5 = p$
Convert to percent.	$150\% = p$ 144 is 150% of 96

Note:
Try It
Exercise:

Problem: 110 is what percent of 88?

Solution:

125%

Note:
Try It
Exercise:

Problem: 126 is what percent of 72?

Solution:

175%

Solve Applications of Percent

Many applications of percent occur in our daily lives, such as tips, sales tax, discount, and mixing solutions. To solve these applications we'll translate to a basic percent equation, just like those we solved in the previous examples in this section. Once you translate the sentence into a percent equation, you know how to solve it.

We will update the strategy we used in our earlier applications to include equations now. Notice that we will translate a sentence into an equation.

Note:
A Strategy to Solve Application Problems

Identify what you are asked to find and choose a variable to represent it.
Write a sentence that gives the information to find it.
Translate the sentence into an equation.
Solve the equation using good algebra techniques.
Check the answer in the problem and make sure it makes sense.
Write a complete sentence that answers the question.

Now that we have the strategy to refer to, and have practiced solving basic percent equations, we are ready to solve percent applications. Be sure to ask yourself if your final answer makes sense—since many of the applications we'll solve involve everyday situations, you can rely on your own experience.

Example:
Exercise:

Problem:

Dezohn and his girlfriend enjoyed a dinner at a restaurant, and the bill was $\$68.50$. They want to leave an 18% tip. If the tip will be 18% of the total bill, how much should the tip be?

Solution:

What are you asked to find?	the amount of the tip
Choose a variable to represent it.	Let $t =$ amount of tip.

Write a sentence that give the information to find it.	The tip is 18% of the total bill.
Translate the sentence into an equation.	$\underbrace{\text{The tip}}_{t}\ \underbrace{\text{is}}_{=}\ \underbrace{18\%}_{0.18}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\$68.50.}_{\$68.50}$
Multiply.	$t = 12.33$
Check. Is this answer reasonable?	If we approximate the bill to \$70 and the percent to 20%, we would have a tip of \$14. So a tip of \$12.33 seems reasonable.
Write a complete sentence that answers the question.	The couple should leave a tip of \$12.33.

Note:
Try It
Exercise:

Problem:

Cierra and her sister enjoyed a special dinner in a restaurant, and the bill was \$81.50. If she wants to leave 18% of the total bill as her tip, how much should she leave?

Solution:

\$14.67

Note:
Try It
Exercise:

Problem:

Kimngoc had lunch at her favorite restaurant. She wants to leave 15% of the total bill as her tip. If her bill was $\$14.40$, how much will she leave for the tip?

Solution:

$\$2.16$

Example:
Exercise:

Problem:

The label on Masao's breakfast cereal said that one serving of cereal provides 85 milligrams (mg) of potassium, which is 2% of the recommended daily amount. What is the total recommended daily amount of potassium?

Nutrition Facts
Serving Size: 1 cup (47g)
Servings Per Container: About 7

Amount Per Serving	Cereal	With Milk
Calories	180	230
Calories from Fat	10	20
	% Daily Value*	
Total Fat 1g	2%	2%
Saturated Fat 0g	0%	0%
Trans Fat 0g		
Polyunsaturated Fat 0.5g		
Monounsaturated Fat 0.5g		
Cholesterol 0mg	0%	0%
Sodium 190mg	8%	11%
Potassium 85mg	2%	8%
Total Carbohydrate 40g	13%	15%
Dietary Fiber 1g	4%	4%
Sugars 8g		
Protein 3g		

Solution:

What are you asked to find?	the total amount of potassium recommended
Choose a variable to represent it.	Let $a =$ total amount of potassium.
Write a sentence that gives the information to find it.	85 mg is 2% of the total amount.
Translate the sentence into an equation.	$\underbrace{85}_{\text{85 mg}} \underbrace{=}_{\text{is}} \underbrace{0.02}_{2\%} \underbrace{\cdot}_{\text{of}} \underbrace{a}_{\text{what}}?$
Divide both sides by 0.02.	

	$\frac{85}{0.02} = \frac{0.02a}{0.02}$
Simplify.	$4,250 = a$
Check: Is this answer reasonable?	Yes. 2% is a small percent and 85 is a small part of 4,250.
Write a complete sentence that answers the question.	The amount of potassium that is recommended is 4250 mg.

Note:
Try It
Exercise:

Problem:

One serving of wheat square cereal has 7 grams of fiber, which is 29% of the recommended daily amount. What is the total recommended daily amount of fiber?

Solution:

24.1 grams

Note:
Try It
Exercise:

Problem:

One serving of rice cereal has 190 mg of sodium, which is 8% of the recommended daily amount. What is the total recommended daily amount of sodium?

Solution:

2,375 mg

Example:
Exercise:

Problem:

Mitzi received some gourmet brownies as a gift. The wrapper said each brownie was 480 calories, and had 240 calories of fat. What percent of the total calories in each brownie comes from fat?

Solution:

What are you asked to find?	the percent of the total calories from fat
Choose a variable to represent it.	Let $p =$ percent from fat.
Write a sentence that gives the information to find it.	What percent of 480 is 240?
Translate the sentence into an	

equation.	$\underbrace{\text{What percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{480}_{480}\ \underbrace{\text{is}}_{=}\ \underbrace{240?}_{240}$
Divide both sides by 480.	$\frac{p \cdot 480}{480} = \frac{240}{480}$
Simplify.	$p = 0.5$
Convert to percent form.	$p = 50\%$
Check. Is this answer reasonable?	Yes. 240 is half of 480, so 50% makes sense.
Write a complete sentence that answers the question.	Of the total calories in each brownie, 50"% is fat.

Note:
Try It
Exercise:

Problem:

The brownie mix Ricardo plans to use says that each brownie will be 190 calories, and 70 calories are from fat. What percent of the total calories are from fat?

Solution:

37%

Example:
Exercise:

Problem:

A technician mixes 30 mL of an acid with water to obtain 75 mL of an acidic solution. What is the percent concentration of the solution? In other words, what percent of acid is in the solution?

Solution:

What are you asked to find?	the percent of the total solution that is acid
Choose a variable to represent it.	Let $p =$ the percent of solution that is acid.
Write a sentence that gives the information to find it.	What percent of 75 mL is 30 mL?
Translate the sentence into an equation.	$\underbrace{\text{What percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{75 mL}}_{75}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{30 mL?}}_{30}$
Divide both sides by 75.	$\frac{p \cdot 75}{75} = \frac{30}{75}$
Simplify.	$p = 0.40$
Convert to percent form.	$p = 40\%$
Check. Is this answer	Yes. 30 is less than half but more

reasonable?	than a third of 75, so 40% makes sense.
Write a complete sentence that answers the question.	40% of the solution is acid.

Note:
Try It
Exercise:

Problem:

If 7.5 grams of a 25 gram solution are alcohol, what is the percent concentration of this solution? In other words, what percent of the solution is alcohol?

Solution:

30%

Find Percent Increase and Percent Decrease

People in the media often talk about how much an amount has increased or decreased over a certain period of time. They usually express this increase or decrease as a percent.

To find the **percent increase**, first we find the amount of increase, which is the difference between the new amount and the original amount. Then we find what percent the amount of increase is of the original amount.

Note:
Find Percent of Increase
Step 1. Find the amount of increase.
$\text{increase} = \text{new amount} - \text{original amount}$
Step 2. Find the percent increase as a percent of the original amount.

Example:
Exercise:

Problem:

In 2011, the California governor proposed raising community college fees from \$26 per unit to \$36 per unit. Find the percent increase. (Round to the nearest tenth of a percent.)

Solution:

What are you asked to find?	the percent increase
Choose a variable to represent it.	Let $p =$ percent.
Find the amount of increase.	$\underbrace{36}_{\text{new amount}} - \underbrace{26}_{\text{original amount}} = \underbrace{10}_{\text{increase}}$
Find the percent increase.	The increase is what percent of the original amount?

Translate to an equation.	$\underbrace{10}_{10}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{what percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{26?}_{26}$
Divide both sides by 26.	$\frac{10}{26} = \frac{26p}{26}$
Round to the nearest thousandth.	$0.385 = p$
Convert to percent form.	$38.5\% = p$
Write a complete sentence.	The new fees represent a 38.5% increase over the old fees.

Note:
Try It
Exercise:

Problem:

In 2011, the IRS increased the deductible mileage cost to 55.5 cents from 51 cents. Find the percent increase. (Round to the nearest tenth of a percent.)

Solution:

8.8%

Note:
Try It
Exercise:

Problem:

In 1995, the standard bus fare in Chicago was $\$1.50$. In 2008, the standard bus fare was $\$2.25$. Find the percent increase. (Round to the nearest tenth of a percent.)

Solution:

50%

Finding the **percent decrease** is very similar to finding the percent increase, but now the amount of decrease is the difference between the original amount and the final amount. Then we find what percent the amount of decrease is of the original amount.

Note:
Find percent decrease

Find the amount of decrease. $\text{decrease} = \text{original amount} - \text{new amount}$
Find the percent decrease as a percent of the original amount.

Example:
Exercise:

Problem:

The average price of a gallon of gas in one city in June 2014 was $3.71. The average price in that city in July was $3.64. Find the percent decrease.

Solution:

What are you asked to find?	the percent decrease
Choose a variable to represent it.	Let $p =$ percent.
Find the amount of decrease.	$\underbrace{3.71}_{\text{original amount}} - \underbrace{3.64}_{\text{new amount}} = \underbrace{0.07}_{\text{decrease}}$
Find the percent of decrease.	The decrease is what percent of the original amount?
Translate to an equation.	$\underbrace{\text{0.07}}_{0.07}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{what percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{3.71?}}_{3.71}$
Divide both sides by 3.71.	$\frac{0.07}{3.71} = \frac{3.71p}{3.71}$

Round to the nearest thousandth.	$0.019 = p$
Convert to percent form.	$1.9\% = p$
Write a complete sentence.	The price of gas decreased 1.9%.

Note:
Try It
Exercise:

Problem:

The population of one city was about $672,000$ in 2010. The population of the city is projected to be about $630,000$ in 2020. Find the percent decrease. (Round to the nearest tenth of a percent.)

Solution:

6.3%

Note:
Try It
Exercise:

Problem:

Last year Sheila's salary was $\$42,000$. Because of furlough days, this year her salary was $\$37,800$. Find the percent decrease. (Round to the nearest tenth of a percent.)

Solution:

10%

Summary

- To translate a percent problem, first write an equation and then solve it. See [link].
- Percents can be used to solve many types of problems. The strategy is to write a sentence that gives the information, translate it to an equation, and solve using algebra. See [link].
- Some percent problems involve finding percent increase or decrease, which is the difference between a new amount and an original amount. See [link].

Homework

Translate and Solve Basic Percent Equations

In the following exercises, translate and solve.

Exercise:

Problem: What number is 45% of 120?

Solution:

54

Exercise:

Problem: What number is 65% of 100?

Exercise:

Problem: What number is 24% of 112?

Solution:

26.88

Exercise:

Problem: What number is 36% of 124?

Exercise:

Problem: 250% of 65 is what number?

Solution:

162.5

Exercise:

Problem: 150% of 90 is what number?

Exercise:

Problem: 800% of $2,250$ is what number?

Solution:

18,000

Exercise:

Problem: 600% of $1,740$ is what number?

Exercise:

Problem: 28 is 25% of what number?

Solution:

112

Exercise:

Problem: 36 is 25% of what number?

Exercise:

Problem: 81 is 75% of what number?

Solution:

108

Exercise:

Problem: 93 is 75% of what number?

Exercise:

Problem: 8.2% of what number is $2.87?

Solution:

$35

Exercise:

Problem: 6.4% of what number is $2.88?

Exercise:

Problem: 11.5% of what number is $108.10?

Solution:

$940

Exercise:

Problem: 12.3% of what number is $\$92.25$?

Exercise:

Problem: What percent of 260 is 78?

Solution:

30%

Exercise:

Problem: What percent of 215 is 86?

Exercise:

Problem: What percent of $1,500$ is 540?

Solution:

36%

Exercise:

Problem: What percent of $1,800$ is 846?

Exercise:

Problem: 30 is what percent of 20?

Solution:

150%

Exercise:

Problem: 50 is what percent of 40?

Exercise:

Problem: 840 is what percent of 480?

Solution:

175%

Exercise:

Problem: 790 is what percent of 395?

Solve Applications of Percents

In the following exercises, solve the applications of percents. Remember to label your answer with units, where appropriate.

Exercise:

Problem:

Geneva treated her parents to dinner at their favorite restaurant. The bill was $\$74.25$. She wants to leave 16% of the total bill as a tip. How much should the tip be?

Solution:

$11.88

Exercise:

Problem:

When Hiro and his co-workers had lunch at a restaurant the bill was $\$90.50$. They want to leave 18% of the total bill as a tip. How much should the tip be?

Exercise:

Problem:

Trong has 12% of each paycheck automatically deposited to his savings account. His last paycheck was $\$2,165$. How much money was deposited to Trong's savings account?

Solution:

$259.80

Exercise:

Problem:

Cherise deposits 8% of each paycheck into her retirement account. Her last paycheck was $\$1,485$. How much did Cherise deposit into her retirement account?

Exercise:

Problem:

One serving of oatmeal has 8 grams of fiber, which is 33% of the recommended daily amount. What is the total recommended daily amount of fiber?

Solution:

24.2 grams

Exercise:

Problem:

One serving of trail mix has 67 grams of carbohydrates, which is 22% of the recommended daily amount. What is the total recommended daily amount of carbohydrates?

Exercise:

Problem:

A bacon cheeseburger at a popular fast food restaurant contains $2,070$ milligrams (mg) of sodium, which is 86% of the recommended daily amount. What is the total recommended daily amount of sodium?

Solution:

2,407 mg

Exercise:

Problem:

A grilled chicken salad at a popular fast food restaurant contains 650 milligrams (mg) of sodium, which is 27% of the recommended daily amount. What is the total recommended daily amount of sodium?

Exercise:

Problem:

The nutrition fact sheet at a fast food restaurant says the fish sandwich has 380 calories, and 171 calories are from fat. What percent of the total calories is from fat?

Solution:

45%

Exercise:

Problem:

The nutrition fact sheet at a fast food restaurant says a small portion of chicken nuggets has 190 calories, and 114 calories are from fat. What percent of the total calories is from fat?

Exercise:

Problem:

Emma gets paid $3,000 per month. She pays $750 a month for rent. What percent of her monthly pay goes to rent?

Solution:

25%

Exercise:

Problem:

Dimple gets paid $3,200 per month. She pays $960 a month for rent. What percent of her monthly pay goes to rent?

Exercise:

Problem:

A lab technician mixes 80 mL of an unknown chemical with 120 mL of water. What is the percent concentration of chemical in the solution? In other words, what percent of the solution is chemical?

Solution:

40%

Exercise:

Problem:

A jar of 250 mL of solution contains 75 mL of sodium. What is the percent concentration of sodium? In other words, what percent of the solution is sodium?

Find Percent Increase and Percent Decrease

In the following exercises, find the percent increase or percent decrease.

Exercise:

Problem:

Tamanika got a raise in her hourly pay, from $15.50 to $17.55. Find the percent increase.

Solution:

13.2%

Exercise:

Problem:

Ayodele got a raise in her hourly pay, from $24.50 to $25.48. Find the percent increase.

Exercise:

Problem:

Annual student fees at the University of California rose from about $4,000 in 2000 to about $9,000 in 2014. Find the percent increase.

Solution:

125%

Exercise:

Problem:

The price of a share of one stock rose from $12.50 to $50. Find the percent increase.

Exercise:

Problem:

According to Time magazine (7/19/2011) annual global seafood consumption rose from 22 pounds per person in 1960 to 38 pounds per person today. Find the percent increase. (Round to the nearest tenth of a percent.)

Solution:

72.7%

Exercise:

Problem:

In one month, the median home price in the Northeast rose from $225,400 to $241,500. Find the percent increase. (Round to the nearest tenth of a percent.)

Exercise:

Problem:

A grocery store reduced the price of a loaf of bread from $2.80 to $2.73. Find the percent decrease.

Solution:

2.5%

Exercise:

Problem:

The price of a share of one stock fell from $8.75 to $8.54. Find the percent decrease.

Exercise:

Problem:

Hernando's salary was $49,500 last year. This year his salary was cut to $44,055. Find the percent decrease.

Solution:

11%

Exercise:

Problem:

From 2000 to 2010, the population of Detroit fell from about 951,000 to about 714,000. Find the percent decrease. (Round to the nearest tenth of a percent.)

Exercise:

Problem:

In one month, the median home price in the West fell from $203,400 to $192,300. Find the percent decrease. (Round to the nearest tenth of a percent.)

Solution:

5.5%

Exercise:

Problem:

Sales of video games and consoles fell from $1,150 million to $1,030 million in one year. Find the percent decrease. (Round to the nearest tenth of a percent.)

Using Proportions to Solve Percent Problems

Learning Objectives

By the end of this section, you will be able to:

- Translate percent problems into proportions
- Solve percent problems using proportions

Translating Percent Problems into Proportions

Previously, we solved percent problems by translating into percent equations and solving by applying the properties of equality we have used to solve linear equations. Some people prefer to solve percent problems by using the proportion method. The proportion method for solving percent problems involves translating a percent problem into a proportion and then solve the proportion using the cross products.

The following is a template for setting up your proportion. Remember that the "amount" is a percentage of the "base" and in place of "percent" we place the percent number (without the % sign).
Equation:

$$\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100}$$

For example, the percent statement "3 is 60% of 5" would translate into the following proportion:
Equation:

$$\frac{3}{5} = \frac{60}{100}$$

If we restate the problem in the words of a proportion, it may be easier to set up the proportion:

Equation:

$$\textit{The amount is to the base as the percent is to one hundred.}$$

First we will practice translating into a percent proportion. Later, we'll solve the proportion.

Example:
Exercise:

Problem: Translate to a proportion. What number is 75% of 90?

Solution:

If you look for the word "of", it may help you identify the base.

Identify the parts of the percent proportion.	$\underbrace{\text{What number}}_{\text{amount}}\ \text{is}\ \underbrace{75\%}_{\text{percent}}\ \text{of}\ \underbrace{90?}_{\text{base}}$
Restate as a proportion.	What number out of 90 is the same as 75 out of 100?
Set up the proportion. Let $n =$ number.	$\frac{n}{90} = \frac{75}{100}$

Note:
Try It
Exercise:

Problem: Translate to a proportion: What number is 60% of 105?

Solution:

$$\frac{n}{105}=\frac{60}{100}$$

Note:
Try It
Exercise:

Problem: Translate to a proportion: What number is 40% of 85?

Solution:

$$\frac{n}{85}=\frac{40}{100}$$

Example:
Exercise:

Problem: Translate to a proportion. 19 is 25% of what number?

Solution:

Identify the parts of the percent proportion.	$\underbrace{19}_{\text{amount}}$ is $\underbrace{25\%}_{\text{percent}}$ of $\underbrace{\text{what number?}}_{\text{base}}$
Restate as a proportion.	19 out of what number is the same as 25 out of 100?
Set up the proportion. Let $n = \text{number}$.	$\frac{19}{n} = \frac{25}{100}$

Note:
Try It
Exercise:

Problem: Translate to a proportion: 36 is 25% of what number?

Solution:

$$\frac{36}{n} = \frac{25}{100}$$

Note:
Try It
Exercise:

Problem: Translate to a proportion: 27 is 36% of what number?

Solution:

$$\frac{27}{n} = \frac{36}{100}$$

Example:
Exercise:

Problem: Translate to a proportion. What percent of 27 is 9?

Solution:

Identify the parts of the percent proportion.	$\underbrace{\text{What percent}}_{\text{percent}}$ of $\underbrace{27}_{\text{base}}$ is $\underbrace{9}_{\text{amount}}$?
Restate as a proportion.	9 out of 27 is the same as what number out of 100?
Set up the proportion. Let $p =$ percent.	$\frac{9}{27} = \frac{p}{100}$

Note:
Try It
Exercise:

Problem: Translate to a proportion: What percent of 52 is 39?

Solution:

$\frac{n}{100} = \frac{39}{52}$

Note:
Try It
Exercise:

Problem: Translate to a proportion: What percent of 92 is 23?

Solution:

$$\frac{n}{100} = \frac{23}{92}$$

Solve Percent Problems Using Proportions

Now that we have translated percent problems into proportions, we are ready to solve the percent problems.

Example:
Exercise:

Problem:

Translate and solve using proportions: What number is 45% of 80?

Solution:

Identify the parts of the percent proportion.	$\underbrace{\text{What number}}_{\text{amount}} \text{ is } \underbrace{45\%}_{\text{percent}} \text{ of } \underbrace{80?}_{\text{base}}$

Restate as a proportion.	What number out of 80 is the same as 45 out of 100?
Set up the proportion. Let $n =$ number.	$\frac{n}{80} = \frac{45}{100}$
Find the cross products and set them equal.	$100 \cdot n = 80 \cdot 45$
Simplify.	$100n = 3,600$
Divide both sides by 100.	$\frac{100n}{100} = \frac{3,600}{100}$
Simplify.	$n = 36$
Write a complete sentence that answers the question.	36 is 45% of 80.
Check if the answer is reasonable.	Yes. 45 is a little less than half of 100 and 36 is a little less than half 80.

Note:
Try It
Exercise:

Problem:

Translate and solve using proportions: What number is 65% of 40?

Solution:

26

Note:
Try It
Exercise:

Problem:

Translate and solve using proportions: What number is 85% of 40?

Solution:

34

In the next example, the percent is more than 100, which is more than one whole. So the unknown number will be more than the base.

Example:
Exercise:

Problem:

Translate and solve using proportions: 125% of 25 is what number?

Solution:

Identify the parts of the percent proportion.	$\underbrace{125\%}_{\text{percent}}$ is $\underbrace{25}_{\text{base}}$ of $\underbrace{\text{what number?}}_{\text{amount}}$
Restate as a proportion.	What number out of 25 is the same as 125 out of 100?
Set up the proportion. Let $n =$ number.	$\frac{n}{25} = \frac{125}{100}$
Find the cross products and set them equal.	$100 \cdot n = 25 \cdot 125$
Simplify.	$100n = 3{,}125$
Divide both sides by 100.	$\frac{100n}{100} = \frac{3{,}125}{100}$
Simplify.	$n = 31.25$
Write a complete sentence that answers the question.	125% of 25 is 31.25.
Check if the answer is reasonable.	Yes. 125 is more than 100 and 31.25 is more than 25.

Note:
Try It
Exercise:

Problem:

Translate and solve using proportions: 125% of 64 is what number?

Solution:

80

Note:
Try It
Exercise:

Problem:

Translate and solve using proportions: 175% of 84 is what number?

Solution:

147

Example:
Exercise:

Problem:

Translate and solve using proportions: What percent of 72 is 9?

Solution:

Identify the parts of the percent proportion.	$\underbrace{\text{What percent}}_{\text{percent}}$ of $\underbrace{72}_{\text{base}}$ is $\underbrace{9?}_{\text{amount}}$
Restate as a proportion.	9 out of 72 is the same as what number out of 100?
Set up the proportion. Let $n =$ number.	$\frac{9}{72} = \frac{n}{100}$
Find the cross products and set them equal.	$72 \cdot n = 100 \cdot 9$
Simplify.	$72n = 900$
Divide both sides by 72.	$\frac{72n}{72} = \frac{900}{72}$
Simplify.	$n = 12.5$
Write a complete sentence that answers the question.	12.5% of 72 is 9.
Check if the answer is reasonable.	Yes. 9 is $\frac{1}{8}$ of 72 and $\frac{1}{8}$ is 12.5%.

Note:
Try It
Exercise:

Problem:

Translate and solve using proportions: What percent of 72 is 27?

Solution:

37.5%

Note:
Try It
Exercise:

Problem:

Translate and solve using proportions: What percent of 92 is 23?

Solution:

25%

Homework

Translate Percent Problems into Proportions

In the following exercises, translate to a proportion.
Exercise:

Problem: What number is 35% of 250?

Solution:

$$\frac{n}{250} = \frac{35}{100}$$

Exercise:

Problem: What number is 75% of 920?

Exercise:

Problem: What number is 110% of 47?

Solution:

$$\frac{n}{47} = \frac{110}{100}$$

Exercise:

Problem: What number is 150% of 64?

Exercise:

Problem: 45 is 30% of what number?

Solution:

$$\frac{45}{n} = \frac{30}{100}$$

Exercise:

Problem: 25 is 80% of what number?

Exercise:

Problem: 90 is 150% of what number?

Solution:

$$\frac{90}{n} = \frac{150}{100}$$

Exercise:

Problem: 77 is 110% of what number?

Exercise:

Problem: What percent of 85 is 17?

Solution:

$\frac{17}{85} = \frac{p}{100}$

Exercise:

Problem: What percent of 92 is 46?

Exercise:

Problem: What percent of 260 is 340?

Solution:

$\frac{340}{260} = \frac{p}{100}$

Exercise:

Problem: What percent of 180 is 220?

Translate and Solve Percent Problems Using Proportions

In the following exercises, translate and solve using proportions.

Exercise:

Problem: What number is 65% of 180?

Solution:

117

Exercise:

Problem: What number is 55% of 300?

Exercise:

Problem: 18% of 92 is what number?

Solution:

16.56

Exercise:

Problem: 22% of 74 is what number?

Exercise:

Problem: 175% of 26 is what number?

Solution:

45.5

Exercise:

Problem: 250% of 61 is what number?

Exercise:

Problem: What is 300% of 488?

Solution:

1464

Exercise:

Problem: What is 500% of 315?

Exercise:

Problem: 17% of what number is $\$7.65$?

Solution:

\$45

Exercise:

Problem: 19% of what number is $\$6.46$?

Exercise:

Problem: $\$13.53$ is 8.25% of what number?

Solution:

\$164

Exercise:

Problem: $\$18.12$ is 7.55% of what number?

Exercise:

Problem: What percent of 56 is 14?

Solution:

25%

Exercise:

Problem: What percent of 80 is 28?

Exercise:

Problem: What percent of 96 is 12?

Solution:

12.5%

Exercise:

Problem: What percent of 120 is 27?

Quantity-Value Tables

Learning Objectives

By the end of this lesson, you will be able to:

- Use a quantity-value table to solve percent problems involving mixtures, as well as other types of percent problems.
- Use the 9-4-4 ratio to convert between food calories and grams of fat, protein, and carbs.

Using a quantity-value table to organize information.
We can use a table to organize data and to help find missing data. The quantity column can be for grams, ounces, milliliters etc. The value column is for the percents for each item. The table below is an example of a **quantity-value table**.

Item	Quantity	Value
Total mixture		100%

Notice that the table lists the items in the mixture and that their percents will add up to 100%.

Example:
Exercise:

Problem:

If 3 oz of boric acid solution are added to 897 oz of water we have a mixture or solution. What is the strength or concentration of this solution?

Solution:

First, we fill in the given information into a quantity-value table:

Item	Quantity	Value
Boric acid	3 oz.	
Water	897 oz.	
Total solution		100%

We can find the missing percents of the acid by first finding the amount of the total solution.

T = 3 oz. + 897 oz. = 900 oz.

Now, we can use the total of 900 oz. to find the percent of boric acid, P. Here, we're answering the question "3 is what percent of 900?"

Thus, using translation, we can write and solve a percent equation to find the percent as follows:

Equation:

$$3 = P \cdot 900$$

$$\frac{3}{900} = \frac{P \cdot 900}{900}$$

$$P \approx 0.0033$$

$$P \approx 0.33\%$$

This solution is a 0.33% boric acid solution, which means the rest (99.67% is water).

Example:
Exercise:

Problem:

Glucose is a mixture of carbon, hydrogen, and oxygen. Here are given numbers in a table. Fill in all the blank spaces.

Item	Quantity	Value
carbon		
hydrogen		6.7%
oxygen	48 gram	53.3%
Total mixture		100%

Solution:

Since we know the percents must add to 100%, we can find the percent of carbon by subtracting 6.7% and 53.3% from 100%: 100% - 6.7%- 53.3% = 40%.

Using the quantity of oxygen and the percent of oxygen, we can determine the quantity of the total mixture:
Equation:

$$\frac{53.3}{100} = \frac{48}{T}$$

$$\frac{53.3T}{53.3} = \frac{4800}{53.3}$$

$$T \approx 90 \text{ g}$$

Using this total, we can now find the quantities of the carbon and hydrogen:

Equation:

$$\frac{40}{100} = \frac{C}{90}$$

$$\frac{100C}{100} = \frac{3600}{100}$$

$$C \approx 36 \text{ g}$$

Equation:

$$\frac{6.7}{100} = \frac{H}{90}$$

$$\frac{100H}{100} = \frac{603}{100}$$

$$H \approx 6 \text{ g}$$

Inserting these values, we now have a completed quantity-value table:

Item	Quantity	Value
carbon	36 gram	40%
hydrogen	6 gram	6.7%
oxygen	48 gram	53.3%
Total mixture	90 gram	100%

Example:
The quantity-value table can also be used for other percent applications.
Exercise:

Problem:

A certain mineral comprises 0.9% of an adult’s body weight. If a person contains 1.44 lb. of the mineral, how much does this person weigh?

Solution:

We can start by organizing our information in a quality-value table:

Item	Quantity	Value
Mineral	1.44 lb.	0.9%
Other		
Total body weight	?	100%

Next, we set up and solve a percent equation to find the total body weight.
Equation:

$$0.009 \cdot T = 1.44$$
$$\frac{0.009 \cdot T}{0.009} = \frac{1.44}{0.009}$$
$$T = 160 \text{ lb.}$$

So, the person weighs 160 pounds.

Mixture Problems With Food Items

Foods are an example of a mixture. Foods are a mixture of fats, proteins, and carbohydrates. These are measured in both grams (for chemical weight) and calories (for energy) as seen on food labels.

See the ratios of calories to gram at the end of this food label.

Nutrition Facts

Serving Size 1 cup (228g)
Servings per Container 2

Amount Per Serving

Calories 280	**Calories from Fat** 120
	% Daily Value*
Total Fat 13g	20%
Saturated Fat 5g	25%
Trans Fat 2g	
Cholesterol 2mg	10%
Sodium 660mg	28%
Total Carbohydrate 31g	10%
Dietary Fiber 3g	0%
Sugars 5g	
Protein 5g	

Vitamin A 4% • Vitamin C 2%
Calcium 15% • Iron 4%

*Percent Daily Values are based on a 2,000-calorie diet. Your daily values may be higher or lower depending on your calorie needs.

	Calories:	**2,000**	**2,500**
Total Fat	Less than	65g	80g
Sat Fat	Less than	20g	25g
Cholesterol	Less than	300mg	300mg
Sodium	Less than	2,400mg	2,400mg
Total Carbohydrate		300g	375g
Fiber		25g	30g

Calories per gram:
Fat 9 • Carbohydrate 4 • Protein 4

Changing between grams and calories can be done with the standard conversion facts; the “9-4-4” numbers.

- One gram of fat = 9 cal.
- One gram of protein = 4 cal.
- One gram of carbs = 4 cal.

The following table is a quantity-value table for food items, where the quantities and values (percents) for fat, protein, and carbs, are organized for both calories and grams, in what we call a double quality-value table.

Quantity grams	Value	Item	Quantity calorie	Value
		Fat		
		Protein		
		Carbs		
	100 %	Total food		100%

Example:
Exercise:

Problem:

If a food has 13 grams of fat, 5 grams of protein, and 31 grams of carbs: let's find the percentages and calories using a double quality-value table.

Solution:

We fill in the given numbers and find all the missing numbers. To find the calories, use the 9-4-4 equivalence numbers. In order to find the calories of each item, we simply multiply the fat grams by 9, the protein grams by 4, and the carbs grams by 4. The total calories are a sum of the calories from fat, protein, and carbs.

Quantity grams	Value	Item	Quantity calorie	Value
13 g		Fat	13(9) = 117 cal.	
5 g		Protein	5(4) = 20 cal.	
31 g		Carbs	31(4) = 124 cal.	
49 g	100 %	Total	261 calories	100%

Now, we can determine the percents of each item for grams and calories.

First, we'll focus on percents for grams. Let F = percent of fat, P = percent of protein, and C = percent of carbs.

$$\frac{F}{100}=\frac{13}{49} \qquad \frac{P}{100}=\frac{5}{49} \qquad \frac{C}{100}=\frac{31}{49}$$

$$\frac{49F}{49}=\frac{1300}{49} \qquad \frac{49P}{49}=\frac{500}{49} \qquad \frac{49C}{49}=\frac{3100}{49}$$

$$F=26.5\% \qquad P=10.2\% \qquad C=63.3\%$$

So, fat is 26.5%, protein is 10.2%, and carbs is 63.3% of the total weight.

Second, let's find the percents for calories. Let F = percent of fat, P = percent of protein, and C = percent of carbs.

$$\frac{F}{100} = \frac{117}{261} \qquad \frac{P}{100} = \frac{20}{261} \qquad \frac{C}{100} = \frac{124}{261}$$

$$\frac{261F}{261} = \frac{11700}{261} \qquad \frac{261P}{261} = \frac{2000}{261} \qquad \frac{261C}{261} = \frac{12400}{261}$$

$$F = 44.8\% \qquad P = 7.7\% \qquad C = 47.5\%$$

So, fat is 44.8%, protein is 7.7%, and carbs is 47.5% of the total amount of calories.

Here is our completed table:

Quantity grams	Value	Item	Quantity calorie	Value
13 g	26.5%	Fat	13(9) = 117 cal.	44.8%
5 g	10.2%	Protein	5(4) = 20 cal.	7.7%
31 g	63.3%	Carbs	31(4) = 124 cal.	47.5%
49 g	100 %	Total	261 calories	100%

Homework

For the following exercises, fill in the given information, using a quantity-value table and find the missing information.

Exercise:

Problem:

21 out of 28 samples were analyzed. What percent were analyzed? What percent still needs to be analyzed?

Solution:

75% have been analyzed and there is 25% more that still needs to be analyzed.

Item	Quantity	Value
Analyzed	21	75
Not analyzed	7	25
Total samples	28	100%

Exercise:

Problem:

If 13 oz. of boric acid solution is added to 1,012 oz. of water we have a mixture or solution. Find the percent of boric acid in this solution.

Exercise:

Problem:

A certain mineral comprises 0.9% of an adult's body weight. If a person contains 1.67 lb. of the mineral, how much does this person weigh?

Solution:

Item	Quantity	Value
Mineral	1.67 lb.	0.9%
Other		
Total body weight	185.6 lb.	100%

Exercise:

Problem:

If 12 mL of sulfuric acid are added to 153 mL of water we have a mixture or solution. What is the strength or concentration of this solution?

Exercise:

Problem:

Use the food label below to fill in a quantity-value chart for both grams and calories.

Nutrition Facts

Serving Size ½ cup (114g)
Servings Per Container 4

Amount Per Serving

Calories 90 Calories from Fat 30

	% Daily Value*
Total Fat 3g	**5%**
Saturated Fat 0g	**0%**
Cholesterol 0mg	**0%**
Sodium 300mg	**13%**
Total Carbohydrate 13g	**4%**
Dietary Fiber 3g	**12%**
Sugars 3g	
Protein 3g	

Vitamin A 80% • Vitamin C 60%

Calcium 4% • Iron 4%

* Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs:

	Calories:	2,000	2,500
Total Fat	Less than	65g	80g
Sat Fat	Less than	20g	25g
Cholesterol	Less than	300mg	300mg
Sodium	Less than	2,400mg	2,400mg
Total Carbohydrate		300g	375g
Dietary Fiber		25g	30g

Calories per gram:
Fat 9 • Carbohydrate 4 • Protein 4

Solution:

Grams	Percent	Item	Calories	Percent
3 g	15.8%	Fat	27 cal.	29.7%
3 g	15.8%	Protein	12 cal.	13.2%
13 g	68.4%	Carbs	52 cal.	57.1%
19 g	100%	Total	91 cal.	100%

Exercise:

Problem:

Use the food label below to fill in a quantity-value chart for both grams and calories.

Nutrition Facts

Serving Size 1 oz. (28g/About 10 crisps)
Servings Per Container 10

Amount Per Serving	
Calories 120	Calories from Fat 30
	% Daily Value*
Total Fat 3g	**5%**
Saturated Fat 0g	**0%**
Trans Fat 0g	
Cholesterol 0mg	**0%**
Sodium 200mg	**8%**
Total Carbohydrate 21g	**7%**
Dietary Fiber 2g	**6%**
Sugars 2g	
Protein 2g	

U.S. Standard Measuring System
In this module, students will learn the U.S. units of measurement in the categories of length, weight/mass, and liquid capacity/volume.

This material will be needed throughout the term. While we will do much more with metrics later in the course, such as conversions and use of the prefix chart, for now you need to memorize the measurements in the three categories of linear, mass or weight, and liquid capacity or volume.

Linear Measurements

The linear category of measurement is for length, width, height, depth, distance, perimeter, etc.

MEMORIZE
You need to memorize the following conversion facts and know the correct abbreviations:

- 12 inches (in.) = 1 foot (ft.)
- 3 feet (ft.) = 1 yard (yd.)
- 5280 feet (ft.) = 1 mile (mi.)

Measurements of Mass or Weight

Mass is the amount of matter in an object and weight is the pull of gravity on it; often both will use the same labels

MEMORIZE
Be sure you memorize the following conversion fact and the correct abbreviations:

1 pound (lb.) = 16 ounces (oz.)

Measurements of Liquid Capacity or Volume

Note: This is not referring to the volume of length × width × height.

MEMORIZE
You need to memorize the following conversion facts and know the correct abbreviations:

- 1 gallon (gal.) = 4 quarts (qt.)
- 1 quart (qt.) = 2 pints (pt.)
- 1 pint (pt.) = 2 cups (c.)
- 1 cup (c.) = 8 fluid ounces (fl. oz.)
- 1 tablespoon (Tbl.) = 3 teaspoons (tsp.)

Homework

After memorizing the conversion facts listed above, fill in the blanks for each problem.

Exercise:

Problem: 1 c. = __________ fl. oz.

Solution:

1 c. = 8 fl. oz.

Exercise:

Problem: 2 pt. = __________ qt.

Exercise:

Problem: 1 mi. = _______________ ft.

Solution:

1 mi. = 5280 ft.

Exercise:

Problem: 1 lb. = __________ oz.

Exercise:

Problem: 1 Tbl. = __________ tsp.

Solution:

1 Tbl. = 3 tsp.

Exercise:

Problem: 1 gal.= _____________ qt.

Exercise:

Problem: 1 ft. = ___________ in.

Solution:

1 ft. = 12 in.

Exercise:

Problem: 1 pt. = ___________ c.

Dimensional Analysis
In this modules, students will learn how to convert units, using the method of dimensional analysis.

Dimensional analysis, also known as unit analysis, is a systematic method for converting from one kind of unit of measurement to another. It is used extensively in chemistry and other health or science related fields. It is an extremely valuable skill to learn, especially when solving complicated application problems.

To convert from one unit of measurement to another (like inches to feet) using dimensional analysis, we use what are called **unit ratios**. Unit ratios are ratios (or fractions) which are equivalent to 1. Remember that a fraction is equivalent to 1 when the numerator and denominator are the same. With unit ratios, the numerator and denominator represent the same measurement, but with different units.

Here are some examples of unit ratios involving units of length:
$$\frac{12in}{1ft}, \frac{1ft}{12in}, \frac{1yd}{3ft}, \frac{3ft}{1yd}$$

These ratios are created from the conversion facts that there are 12 inches in a foot and 3 feet in a yard.

Notice that each conversion fact above gives two possible unit ratios. Deciding which one to use will depend on where units need to be placed in order to cross cancel. In dimensional analysis, we multiply ratios together and cancel common units the same way we cancel common factors when multiplying fractions.

Refer to the U.S. Standard Measuring System for conversion facts that will be used in the following examples and homework problems.

Example:
Exercise:

Problem: Convert 60 inches into feet.

Solution:

The inches cross cancel, like common factors.

$$\frac{60\cancel{in}}{1}\cdot\frac{1\,ft}{12\cancel{in}} = \frac{60\cdot 1\,ft}{1\cdot 12} = \frac{60\,ft}{12} = 5\,ft$$

Always start by writing what needs to be converted over 1.

Multiply by the unit ratio which has *feet* on top and *inches* on bottom.

Multiply what's left, including the units, which are *feet*. Then simplify.

Notice that we start by writing what we are converting as a ratio, by placing it over a 1. This is similar to writing a whole number as a fraction when we want to multiply it times a fraction.

Example:
Exercise:

Problem: Convert 1760 feet into miles.

Solution:

The feet cross cancel, like common factors.

$$\frac{1760\,\cancel{ft}}{1}\cdot\frac{1mi}{5280\,\cancel{ft}} = \frac{1760mi}{5280} = \frac{1}{3}mi \text{ or } 0.\overline{3}mi$$

Always start by writing what needs to be converted over 1.

Multiply by the unit ratio which has *miles* on top and *feet* on bottom.

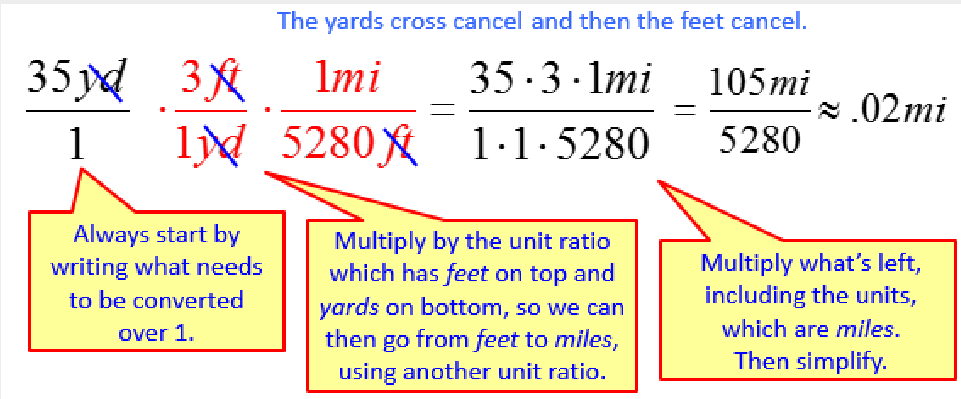
Multiply what's left, including the units, which are *miles*. Then simplify.

In the two previous examples, only one unit ratio was needed to make the conversion. However, in most cases, we need more than one unit ratio. In those situations, we have to consider the conversion facts that are available and then make a plan.

Example:
Exercise:

Problem: Convert 35 yards into miles.

Solution:



Since there are no conversion facts on our list of facts to memorize that relate yards and miles, we had to use two unit ratios. First, we converted yards to feet, using the fact that there are 3 feet in a yard. Then, we converted the feet to miles, using the fact that there are 5280 feet in a mile.

Note:
Try It
Exercise:

Problem: Use dimensional analysis to convert 0.75 miles into inches.

Solution:

$$\frac{0.75\,\cancel{mi}}{1}\cdot\frac{5280\,\cancel{ft}}{1\,\cancel{mi}}\cdot\frac{12in}{1\,\cancel{ft}}=47{,}520\,in$$

Example:

Convert 3 pounds into ounces.

$$\frac{3\cancel{lb}}{1}\cdot\frac{16oz}{1\cancel{lb}}=(3)(16)oz=48oz$$

Always start by writing what needs to be converted over 1.

Multiply by the unit ratio which has *oz* on top and *lb* on bottom.

Multiply what's left, including the units, which are *ounces*. Then simplify.

Example:

Exercise:

Problem: How many gallons are in 34 quarts?

Solution:

$$\frac{34\ \cancel{\text{qt}}}{1}\cdot\frac{1\ \text{gal}}{4\ \cancel{\text{qt}}}=\frac{34\ \text{gal}}{4}=8.25\ \text{gal}$$

Always start by writing what needs to be converted over 1.

Multiply by the unit ratio which has *gal* on top and *qt.* on bottom, so the *qt.* will cross cancel.

Multiply what's left, including the units, which are *gallons*. Then simplify.

Example:
Exercise:

Problem: How many ounces are in 7 gallons?

Solution:

$$\frac{7\cancel{\text{gal}}}{1}\cdot\frac{4\ \cancel{\text{qt}}}{1\ \cancel{\text{gal}}}\cdot\frac{2\ \cancel{\text{pt}}}{1\ \cancel{\text{qt}}}\cdot\frac{2\ \cancel{\text{cups}}}{1\ \cancel{\text{pt}}}\cdot\frac{8\ \text{oz}}{1\ \cancel{\text{cup}}}=(7)(4)(2)(2)(8)\text{oz}=896\ \text{oz}$$

Always start by writing what needs to be converted over 1.

Multiply by the unit ratios needed to make all units cancel, except for ounces, which is the unit we want to end up with.

Multiply what's left, including the units, which are *ounces*. Then simplify.

Notice that several unit ratios are needed here. The first one has gallons in the denominator so that the gallons will cross cancel. Then, a second one has quarts in the denominator so that the quarts will cross cancel. This process continues until all units cancel except for the units that you are converting to.

Note:
Try It
Exercise:

Problem:

Use dimension analysis to answer the question: How many quarts are there in 100 cups?

Solution:

$$\frac{100\ \cancel{c}}{1}\cdot\frac{1\ \cancel{pt}}{2\ \cancel{c}}\cdot\frac{1qt}{2\ \cancel{pt}}=25qt$$

Homework

Use dimensional analysis to complete the following homework exercises. Even though only final answers will be shown in the given solutions, be sure to write down the process, just as it has been shown to you.

Exercise:

Problem: Convert 22.5 yards to inches.

Solution:

810 inches

Exercise:

Problem: Convert 3 gallons to pints.

Exercise:

Problem: How many pounds are in 500 ounces?

Solution:

31.25 lb.

Exercise:

Problem: How many miles are in 26,928 ft?

Exercise:

Problem: Convert 1000 yards into miles.

Solution:

Approximately, 0.57 miles.

Exercise:

Problem: Convert 2.5 quarts to fl. oz.

Metric System
In this module, students will learn about the metric system, including the most commonly used prefixes.

The metric system of measurement uses a starting base for each category (linear, mass/weight, and liquid capacity/volume). The base with a prefix represents either a multiple of a subdivision of the base unit.

Subdivisions are parts or fractions of the base unit and the most common subdivisions are represented using the following prefixes:

- One tenth of the base unit is represented by **"deci"**. It takes 10 deci to make a whole.
- One hundredth of a base unit is represented by **"centi"**. It takes 100 centi to make a whole.
- One thousandth of a base unit is represented by **"milli"**. It takes 1000 milli to make a whole.

Multiples means more than one of the base unit. The most common multiples are represented using the following prefixes:

- Ten of the base unit is represented by **"deka"**. One deka is 10 times a whole.
- One hundred of the base unit is represented by **"hecto"**. One hecto is 100 times a whole.
- One thousand of the base unit is represented by **"kilo"**. One kilo is 1000 times a whole.

Linear Measurements

The linear category is for length, width, height, depth, distance, perimeter, etc.

The base unit in the linear category is the **meter** (abbreviated as m). This is about the size of a yardstick or height of a door knob from the floor.

The subdivision of centimeter is about the width of your little finger.
The subdivision of millimeter is about the thickness of a pencil lead.

MEMORIZE
You need to memorize these conversion facts and know the correct abbreviations:

- 1 meter (m) = 100 centimeters (cm)
- 1 meter (m) = 1000 millimeters (mm)
- 1 centimeter (cm) = 10 millimeters (mm)

Measurements of Weight or Mass

Mass is the amount of matter in an object and weight is the pull of gravity on it; often both use the same labels.

The base unit for this category is the **gram** (abbreviated as g), which is about the weight of a paper clip. The multiple of kilogram is about 2 pounds.

MEMORIZE
You need to memorize the following conversion facts and know the correct abbreviations:

- 1 kilogram (kg) = 1000 grams (g)
- 1 gram (g) = 1000 milligrams (mg)

Measurements of Liquid Capacity or Volume

The base unit for this category is the **liter** (abbreviated as L), which is about a quart.

MEMORIZE
You need to memorize the following conversion facts and know the correct abbreviations:

- 1 liter (L) = 1000 milliliter (mL)

- 1 liter (L) = 10 deciliters (dL)
- 1 deciliter (dL) = 100 milliliters (mL)

Note that you may see a lower case l used to represent liters. However, in many fonts, it is difficult to distinguish it from the number 1 or the capital letter I, so using a capital L for liters is preferred.

Note: While any prefix can be used with any base,

- the most common prefixes for the meter are: km, cm, and mm;
- the most common prefixes for the weight or mass category are kg and mg;
- the most common prefixes for the liquid capacity or volume are dL (deciliter) and mL (milliliter).

Homework

After memorizing the conversion facts above, complete these problem by filling in the blank.

Exercise:

Problem: 1 m = ___________ mm

Solution:

1 m = 1000 mm

Exercise:

Problem: 1 kg = ___________ g

Exercise:

Problem: 1 dL = ________ mL

Solution:

1 dL = 100 mL

Exercise:

Problem: 1 m = __________ cm

Exercise:

Problem: 1 g = __________ mg

Solution:

1 g = 1000 mg

Exercise:

Problem: 1 L = _________ dL

Exercise:

Problem: 1 cm = _________ mm

Solution:

1 cm = 10 mm

Exercise:

Problem: 1 L = ___________ mL

In the following exercises, match each base unit to its category:
Exercise:

Problem: The base unit for the linear category is __________.

a. gram
b. meter
c. liter

Solution:

b

Exercise:

Problem:

The base unit for the liquid capacity or volume category is __________.

a. gram
b. meter
c. liter

Converting Between Metric Measurements

Learning Objectives

By the end of this lesson, you will:

- be more familiar with some of the advantages of the base ten number system,
- have reviewed the prefixes of the metric measures,
- be able to convert from one metric unit of measure to another metric unit of measure, by moving the decimal.

The Advantages of the Base Ten Number System

The metric system of measurement takes advantage of our base ten number system. The advantage of the metric system over the United States system is that in the metric system it is possible to convert from one unit of measure to another simply by multiplying or dividing the given number by a power of 10. This means we can make a conversion simply by moving the decimal point to the right or the left.

Prefixes

Common units of measure in the metric system are the meter (for length), the liter (for volume), and the gram (for mass). To each of the units can be attached a prefix. The **metric prefixes** along with their meaning are listed below.
Metric Prefixes

- **kilo**thousand
- **deci**tenth
- **hecto**hundred
- **centi**hundredth
- **deka**ten
- **milli**thousandth

For example, if length is being measured,

1 kilometer is equivalent to 1000 meters.

1 centimeter is equivalent to one hundredth of a meter.
1 millimeter is equivalent to one thousandth of a meter.

Conversion from One Unit to Another Unit

Let's note three characteristics of the metric system that occur in the metric table of measurements.

1. In each category, the prefixes are the same.
2. We can move from a *larger to a smaller* unit of measure by moving the decimal point to the *right*.
3. We can move from a *smaller to a larger unit* of measure by moving the decimal point to the *left*.

The following table provides a summary of the relationship between the basic unit of measure (meter, gram, liter) and each prefix, and how many places the decimal point is moved and in what direction.

kilo hecto deka unit deci centi milli

Basic Unit to Prefix		Move the Decimal Point
unit to deka	1 to 10	1 place to the left
unit to hector	1 to 100	2 places to the left
unit to kilo	1 to 1,000	3 places to the left
unit to deci	1 to 0.1	1 place to the right
unit to centi	1 to 0.01	2 places to the right
unit to milli	1 to 0.001	3 places to the right

Conversion Table

Listed below, in the unit conversion table, are some of the common metric units of measure.

Unit Conversion Table		
Length	$1\ \text{kilometer}\ \ (\text{km}) = 1{,}000\ \text{meters}\ \ (m)$	$1{,}000 \times 1\text{m}$
	$1\ \text{hectometer}\ \ (\text{hm}) = 100\ \text{meters}$	$100 \times 1\text{m}$
	$1\ \text{dekameter}\ \ (\text{dam}) = 10\ \text{meters}$	$10 \times 1\text{m}$
	1 meter (m)	$1 \times 1\text{m}$
	$1\ \text{decimeter}\ \ (\text{dm}) = \frac{1}{10}\ \text{meter}$	$.1 \times 1\text{m}$
	$1\ \text{centimeter}\ \ (\text{cm}) = \frac{1}{100}\ \text{meter}$	$.01 \times 1\text{m}$
	$1\ \text{millimeter}\ \ (\text{mm}) = \frac{1}{1{,}000}\ \text{meter}$	$.001 \times 1\text{m}$
Mass	$1\ \text{kilogram}\ \ (\text{kg}) = 1{,}000\ \text{grams}\ \ (g)$	$1{,}000 \times 1\text{g}$
	$1\ \text{hectogram}\ \ (\text{hg}) = 100\ \text{grams}$	$100 \times 1\text{g}$
	$1\ \text{dekagram}\ \ (\text{dag}) = 10\ \text{grams}$	$10 \times 1\text{g}$
	$1\ \text{gram}\ \ (\text{g})$	$1 \times 1\text{g}$
	$1\ \text{decigram}\ \ (\text{dg}) = \frac{1}{10}\ \text{gram}$	$.1 \times 1\text{g}$
	$1\ \text{centigram}\ \ (\text{cg}) = \frac{1}{100}\ \text{gram}$	$.01 \times 1\text{g}$

	$1 \quad \text{milligram} \quad (\text{ mg}) = \frac{1}{1{,}000} \quad \text{gram}$	$.001 \times 1\text{g}$
Volume	$1 \quad \text{kiloliter} \quad (\text{ kL}) = 1{,}000 \ \text{liters} \quad (\ L)$	$1{,}000 \times 1\text{L}$
	$1 \quad \text{hectoliter} \quad (\text{ hL}) = 100 \ \text{liters}$	$100 \times 1\text{L}$
	$1 \quad \text{dekaliter} \quad (\text{ daL}) = 10 \ \text{liters}$	$10 \times 1\text{L}$
	1 liter (L)	$1 \times 1\text{L}$
	$1 \quad \text{deciliter} \quad (\text{ dL}) = \frac{1}{10} \quad \text{liter}$	$.1 \times 1\text{L}$
	$1 \quad \text{centiliter} \quad (\text{ cL}) = \frac{1}{100} \quad \text{liter}$	$.01 \times 1\text{L}$
	$1 \quad \text{milliliter} \quad (\text{ mL}) = \frac{1}{1{,}000} \quad \text{liter}$	$.001 \times 1\text{L}$

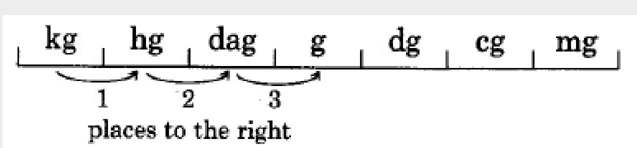
Converting Metric Units
To convert from one metric unit to another metric unit:

1. Determine the location of the original number on the metric scale (pictured in each of the following examples).
2. Move the decimal point of the original number in the same direction and same number of places as is necessary to move to the metric unit you wish to convert to.

We can also convert from one metric unit to another using dimensional analysis.

Example:
Convert 3 kilograms to grams.

a. 3 kg can be written as 3.0 kg. Then,



$3.0\text{ kg} = 3\,\underbrace{000}_{1\,2\,3}.\text{ g}$

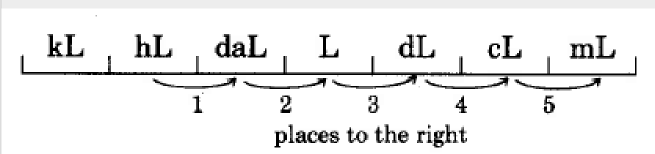
Thus, $3\text{kg} = 3{,}000$ g.

b. We can also use dimensional analysis to make this conversion.

Since we are converting to grams, and 1000 g = 1 kg, we choose the unit ratio $\frac{1{,}000 \quad \text{g}}{1 \quad \text{kg}}$ since grams is in the numerator.

$$\frac{3\ \cancel{\text{kg}}}{1} \cdot \frac{1000\text{ g}}{1\ \cancel{\text{kg}}} = 3{,}000\text{ g}$$

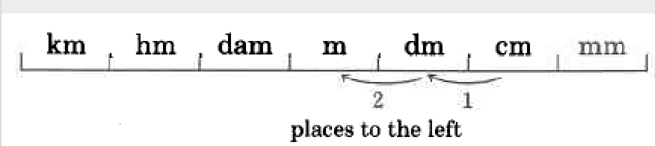
Example:
Convert 67.2 hectoliters to milliliters.



$67.2\text{ hL} = 67\,\underbrace{20000}_{1\,2\,3\,4\,5}.\text{ mL}$

Thus, $67.2\text{ hL} = 6{,}720{,}000\text{ mL}$.

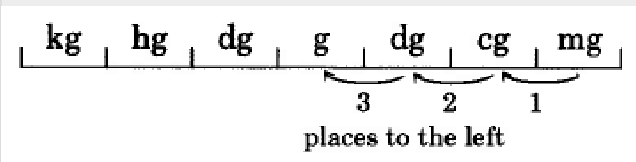
Example:
Convert 100.07 centimeters to meters.



$100.07\text{ cm} = 1.\underbrace{00}_{2\,1}07\text{ m}$

Thus, 100.07 cm = 1.0007 m.

Example:
Convert 0.16 milligrams to grams.



$$0.16 \text{ mg} = 0.\underbrace{000}_{3\,2\,1}16 \text{ g}$$

Thus, 0.16 mg = 0.00016 g.

Note:
Try It
Exercise:

Problem: Convert 411 kilograms to grams.

Solution:

411,000 g

Note:
Try It
Exercise:

Problem: Convert 5.626 liters to centiliters.

Solution:

562.6 cL

Note:
Try It
Exercise:

Problem: Convert 80 milliliters to kiloliters.

Solution:

0.00008 kL

Note:
Try It
Exercise:

Problem: Convert 150 milligrams to centigrams.

Solution:

15 cg

Homework

Make each conversion.
Exercise:

Problem: 87 m to cm

Solution:

8,700 cm

Exercise:

Problem: 905 L to mL

Exercise:

Problem: 16,005 mg to g

Solution:

16.005 g

Exercise:

Problem: 48.66 L to dL

Exercise:

Problem: 11.161 kL to L

Solution:

11,161 L

Exercise:

Problem: 521.85 cm to mm

Exercise:

Problem: 1.26 dag to dg

Solution:

126 dg

Exercise:

Problem: 99.04 dam to cm

Exercise:

Problem: 0.51 kL to daL

Solution:

51 daL

Exercise:

Problem: 0.17 kL to daL

Exercise:

Problem: 0.05 m to dm

Solution:

0.5 dm

Exercise:

Problem: 0.001 km to mm

Exercise:

Problem: 8.106 hg to cg

Solution:

81,060 cg

Exercise:

Problem: 17.0186 kL to mL

Exercise:

Problem: 3 cm to m

Solution:

0.03 m

Exercise:

Problem: 9 mm to m

Exercise:

Problem: 4 g to mg

Solution:

4,000 mg

Exercise:

Problem: 2 L to kL

Converting Between the Two Systems of Measurement
In this module, students will learn some conversion facts that compare units from the U.S. Standard and Metric systems of measurement, and use dimensional analysis to convert between the two systems.

Learning Objectives

By the end of this section, you will be able to:

- Memorize a list of conversion facts relating U.S. Standard measurements to Metric measurements.
- Use dimensional analysis to convert between the two systems of measurement.

Note:
MEMORIZE
You should memorize the following conversion facts relating the two systems of measurement:

- 1 in. = 2.54 cm
- 1 km = 0.62 mi.
- 1 kg = 2.2 lb.
- 1 qt. = 946 mL
- 1 tsp = 5 mL
- 1 fl. oz. = 30 mL
- 1 cup = 240 mL

In this course, you should use only these conversions facts, in addition to the ones you were asked to memorize in the U.S. Standard Measuring System and Metric System lessons.

Example:

Exercise:

Problem: Use dimensional analysis to convert 150 mL into fl. oz.

Solution:

$$\frac{150\,\cancel{\text{mL}}}{1}\cdot\frac{1\text{ fl. oz.}}{30\,\cancel{\text{mL}}}=5\text{ fl. oz.}$$

Notice how we start by writing what we are converting (150 mL) as a ratio. Then we multiply by the unit ratio that comes from the fact that 1 fl. oz. = 30 mL, written so that the mL will cross cancel.

Example:
Exercise:

Problem: Use dimensional analysis to convert 5 lb. to grams.

Solution:

$$\frac{5\,\cancel{\text{lb.}}}{1}\cdot\frac{1\,\cancel{\text{kg}}}{2.2\,\cancel{\text{lb.}}}\cdot\frac{1000\text{ g}}{1\,\cancel{\text{kg}}}=\text{about }2273\text{ g.}$$

We don't have any conversion facts on the list that relate pounds to grams directly. However, we do have the facts that there are about 2.2 lb in a kg and there are 1000 g in a kg. So, we need two unit ratios, using those facts, writing the unit ratios in such a way that the lb will cross cancel and then the kg will cross cancel.

Example:
Exercise:

Problem: Use dimensional analysis to convert 2 liters to fl. oz.

Solution:

$$\frac{2\ \cancel{L}}{1}\cdot\frac{1000\ \cancel{mL}}{1\ \cancel{L}}\cdot\frac{1\ \text{fl. oz.}}{30\ \cancel{mL}} = \text{about } 67\ \text{fl. oz.}$$

Example:
Exercise:

Problem: Use dimensional analysis to convert 8 kg to lb.

Solution:

$$\frac{8\ \cancel{kg}}{1}\cdot\frac{2.2\ \text{lb.}}{1\ \cancel{kg}} = 17.6\ \text{lb.}$$

Note:
Try It
Exercise:

Problem: Use dimensional analysis to convert 50 fl. oz. to L.

Solution:

$$\frac{50\ \cancel{\text{fl. oz.}}}{1}\cdot\frac{30\ \cancel{mL}}{1\ \cancel{\text{fl. oz.}}}\cdot\frac{1\ L}{1000\ \cancel{mL}} = 1.5\ L$$

Homework

Use dimensional analysis and the memorized conversion facts to make the following conversions. Be sure to show all the steps as shown in the examples.

Exercise:

Problem: Convert 4 ft to cm.

Solution:

121.92 cm

Exercise:

Problem: Convert 5 gal to mL.

Exercise:

Problem: Convert 65 cm to in.

Solution:

25.6 in.

Exercise:

Problem: Convert 20 mL to Tbl.

Exercise:

Problem: Convert 22 km to ft.

Solution:

About 72,000 ft.

Exercise:

Problem: Convert 10 mi to m.

Exercise:

Problem: Convert 7.5 kg to oz.

Solution:

264 oz.

Exercise:

Problem: Convert 5 oz. to mg.

Exercise:

Problem:

A canned beverage typically contains 12 fl. oz. How many mL of beverage does one of these typical cans contain?

Solution:

About 360 mL

Exercise:

Problem: A child weighs 55 pounds. What is this child's weight in kg?

Exercise:

Problem:

Rojeet is traveling in London in a rental car. After filling the gas tank, he sees that it took 64 L to fill the tank. How many gallons of gasoline is this? Round to the nearest gallon.

Solution:

About 17 gallons.

Exercise:

Problem:

Marjorie is baking a cake and the recipe calls for three-fourths of a cup milk. How many mL of milk does this recipe call for?

Medical Abbreviations and Systems

Learning Objectives

By the end of this lesson, you will

- Know different forms of a drug and their abbreviations.
- Know different routes for a drug to be administered and their abbreviations.
- Know different times for how often a drug is to be administered and their abbreviations.
- Understand how to use a 24-hour clock (or military time).
- Know the apothecary and household systems of measurement.

You will need to know these abbreviations for homework and for tests. Even if you are not studying to be in the health field, it will be good for you to know these abbreviations for your own family's medical needs.

Forms (the form of the drug)

- **caps** is the abbreviation for **capsules**
- **sol** is the abbreviation for **solution**
- **tabs** is the abbreviation for **tablets**

Routes (how the drug gets into the body)

- **I.M.** is the abbreviation for **intramuscular**
- **I.V.** is the abbreviation for **intravenously**
- **P.O.** is the abbreviation for **by mouth**
- **S.C.** is the abbreviation for **subcutaneously**, which is under the skin.

Knowing the route can help us decide if the amount of medicine we calculate is reasonable. I.M. and S.C. would be administered with syringes so the amount would be small such as drops or a few mL while an I.V. amount could be about a liter as it would drip into the vein over a length of time. P.O. is by mouth so it would be just a few tablets or capsules.

Time (how often the drug is to be administered)

- **bid** is twice a day, while awake
- **tid** is three times a day, while awake
- **qid** is four times a day, while awake
- **q12h** is every 12 hours, around the clock
- **q8h** is every 8 hours, around the clock
- **q6h** is every 6 hours, around the clock
- **q4h** is every 4 hours, around the clock
- **q3h** is every 3 hours, around the clock
- **q2h** is every 2 hours, around the clock

The 24-hour Clock

The 24-hour clock is a time keeping convention where the day runs from midnight to midnight and is split into 24 hours, from hour 0 to hour 23. It is the most common system in use in the world and is the international standard notation of time.

In the U.S. and Canada the 12-hour AM/PM format is still more commonly used. In these countries the 24-hour format is called 'Military time'.

Here is a web site with diagrams and practice:
https://www.mathsisfun.com/time.html

Changing AM time to military time is just a change in notation.

1:35 am is written as 01:35.

11:45 am is written as 11:45.

12:05 am is just after midnight and is written as 00:05

Changing PM time to military uses addition of 12 hours and a change in notation.

3:15 pm is 15:15. (since 3 + 12 = 15).

8:00 pm is 20:00. (since 8 + 12 = 20)

10:25 pm is 22:25 (since 10 + 12 = 22)

12:30 pm is just after noon and is written 12:30. (since 0 + 12 = 12)

Body sizes used for dosage

Sometimes dosages are based on the size of the patient, either by weight in kg or by the BSA, which is the body surface area.

BSA is the amount of skin coverage on a person where interactions may occur between drugs and the inner or outer environment. This is measured in square meters. It comes from a calculation that uses the height and the weight of a person. We won't do the calculation in this course, but we will estimate the BSA of a person.

- An adult's BSA is about 2 – 3 sq. meters;
- A small adult's or teenager's BSA is about 1.5 sq. meters;
- A child's BSA is about 1 sq. meter;
- The BSA for infants is less than 0.5 square meters.

You need to memorize these 4 estimates for this class. The numbers are just estimates, rounded to an easy number to memorize, but they are fairly accurate. Rarely is an adult over 3 square meters while infants could be 0.2 square meters.

Note:BSA is not the same as BMI, the body mass index. While this number also uses height and weight, it is an estimation of body fat and doesn't include a label.

Here is an online calculator if you want to find your own Body Surface Area: http://www.medcalc.com/body.html

Older measurement systems

The **Household system** is used for giving some medicines at home and is the same as the American system: teaspoon, tablespoon, fl. oz., etc. However, an additional measurement is the **drop** written as **gtt**.

Medical orders are given in metrics and may be estimated in household units for ease of administering in the home. The equivalents are not very exact so they are only used for estimates.

The **Apothecary system** is not the main system used in medical fields today, but is still seen on tools and in records. The apothecary conversion facts and their symbols don't need to be memorized, but you need to recognize their abbreviations and be familiar with the numbers in the conversion facts.

We will only do limited work with this system and then only with these few conversion facts:

- The **minim**, abbreviated as **min** or $\mathbf{M_x}$ is a unit of volume and is about a drop (or 1 **gtt**).
- The **fluidram** abbreviated as **fl. dr.** or fʒ, and sometimes called dram for short, is for volume of liquid and is about 3/4 of a teaspoon. There are also about 8 fluidrams in 1 fluid ounce (fl oz).
- **Grains**, abbreviated as **gr** is for weight as with aspirin and other powders, and 1 grain is about 60 mg. (Be sure to use g for grams and gr for grains.)

Here is a link to some measurement conversion tables involving the apothecary system, for further reference.

Example:
Exercise:

Problem:

How many ʒ (drams) are in 20 grains if one dram = 60 grains?

Solution:

$$\frac{20\ \cancel{gr}}{1}\cdot\frac{1\ \text{dram}}{60\ \cancel{gr}}=\boxed{\frac{1}{3}\ \text{dram (or }\approx 0.33\ \text{dram)}}$$

Note:
Try It
Exercise:

Problem:

How many minims are in 2.5 fl. oz. given 1 fl. oz. = 8 fl. dram and 1 fl. dram = 60 minims?

Solution:

$$\frac{2.5\ \cancel{\text{fl. oz.}}}{1}\cdot\frac{8\ \cancel{\text{fl. dram}}}{1\ \cancel{\text{fl. oz.}}}\cdot\frac{60\ \text{minims}}{1\ \cancel{\text{fl. dram}}}=\boxed{1200\ \text{minims}}$$

Example:
Exercise:

Problem: How many grains are in 2.4 g?

Solution:

Note that since we only have a conversion fact given that relates grains to milligrams, we will first need to convert the grams to milligrams, using the fact that there are 1000 mg in one gram.

$$\frac{2.4\ \cancel{g}}{1}\cdot\frac{1000\ \cancel{mg}}{1\ \cancel{g}}\cdot\frac{1\ gr}{60\ \cancel{mg}}=\frac{2400\ gr}{60}=\boxed{40\ gr}$$

Homework

For the following exercises, write the meanings of the given abbreviations.

Exercise:

Problem: P.O.

Solution:

The medicine is to be administered by mouth.

Exercise:

Problem: tid

Exercise:

Problem: q2h

Solution:

The medicine needs to be administered every 2 hours, around the clock.

Exercise:

Problem: tab

Exercise:

Problem: qid

Solution:

The medicine needs to be administered four times a day, during wake times.

Exercise:

Problem: gtt

Exercise:

Problem: I.V.

Solution:

The medicine is to be administered intravenously.

For the following exercises, convert between AM/PM time and time using the 24-hour clock.

Exercise:

Problem: 05:30

Exercise:

Problem: 2:15 pm

Solution:

14:15

Exercise:

Problem: 4:05 pm

Exercise:

Problem: 17:45

Solution:

5:45 pm

For the following exercises, estimate whether the BSA belongs to an adult, a small adult, a child, or an infant.

Exercise:

Problem: BSA = 1.6 m^2

Exercise:

Problem: BSA = 0.35 m^2

Solution:

An infant

For the following exercise, use dimensional analysis to make the conversions.

Exercise:

Problem:

How many grains are in 1.5 ounce if 1 ounce = 8 drams and 1 dram = 60 grains?

Exercise:

Problem: Convert 6000 mg to drams, given 1 dram = 4 grams.

Solution:

1.5 drams

Exercise:

Problem:

How many fluid drams are in 10 mL? Use 4 mL per 1 fl. dram.

Exercise:

Problem:

How many minims are in half a teaspoon if there are 16 minims in one milliliter?

Solution:

40 minim

Exercise:

Problem: How many grams are in 36 grains?

Exercise:

Problem: Convert 4 tablespoons to fluid drams.

Solution:

16 fluid drams.

Volume of Solids and Density

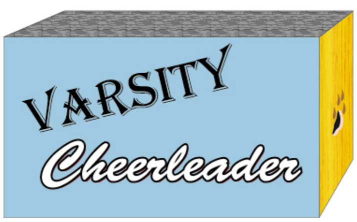
Learning Objectives

By the end of this section, you will be able to:

- Find volume rectangular solids
- Find volume of cylinders
- Solve density problems using the density formula, a proportion, or dimensional analysis

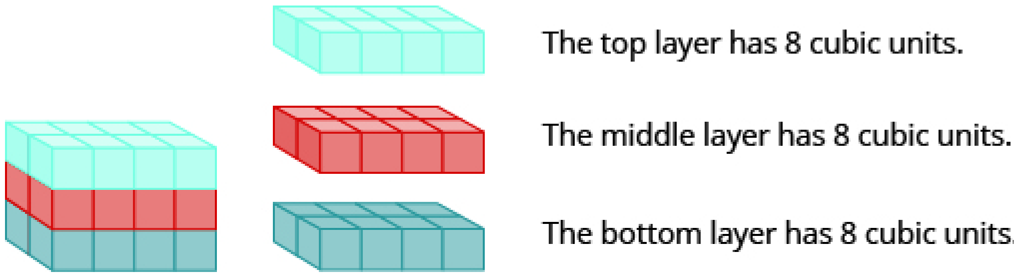
Find Volume of Rectangular Solids

A cheerleading coach is having the squad paint wooden crates with the school colors to stand on at the games. The amount of space inside the crate is the volume, a cubic measure.



This wooden crate is in the shape of a rectangular solid.

Each crate is in the shape of a **rectangular solid**. Its dimensions are the length, width, and height. The rectangular solid shown in [link] has length 4 units, width 2 units, and height 3 units. Can you tell how many cubic units there are altogether? Let's look layer by layer.



Breaking a rectangular solid into layers makes it easier to visualize the number of cubic units it contains. This 4 by 2 by 3 rectangular solid has 24 cubic units.

Altogether there are 24 cubic units. Notice that 24 is the $\text{length} \times \text{width} \times \text{height}$.

$$\underbrace{V}_{24} \; \underbrace{=}_{=} \; \underbrace{L}_{4} \; \underbrace{\cdot}_{\cdot} \; \underbrace{W}_{2} \; \underbrace{\cdot}_{\cdot} \; \underbrace{H}_{3}$$

The volume, V, of any rectangular solid is the product of the length, width, and height.

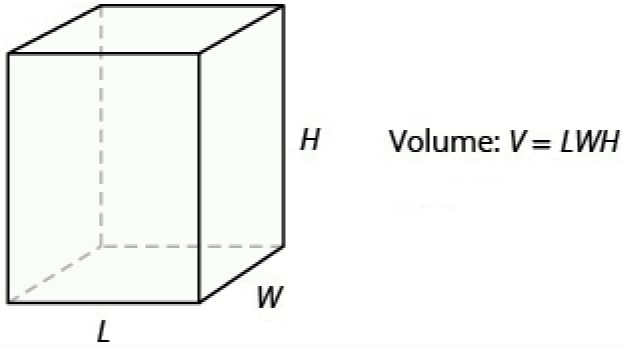
Equation:

$$V = LWH$$

Note:

Volume of a Rectangular Solid

For a rectangular solid with length L, width W, and height H :

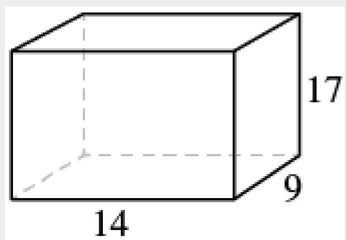


Example:
Exercise:

Problem:

For a rectangular solid with length 14 cm, height 17 cm, and width 9 cm, find the volume.

Solution:

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the volume of the rectangular solid
Step 3. **Name.** Choose a	Let V = volume

variable to represent it.	
Step 4. **Translate.** Write the appropriate formula. Substitute.	$V = LWH$ $V = 14 \cdot 9 \cdot 17$
Step 5. **Solve** the equation.	$V = 2{,}142$
Step 6. **Check** We leave it to you to check your calculations.	
Step 7. **Answer** the question.	The volume is $2{,}142$ cubic centimeters.

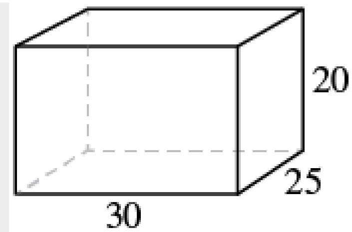
Example:
Exercise:

Problem:

A rectangular crate has a length of 30 inches, width of 25 inches, and height of 20 inches. Find its volume.

Solution:

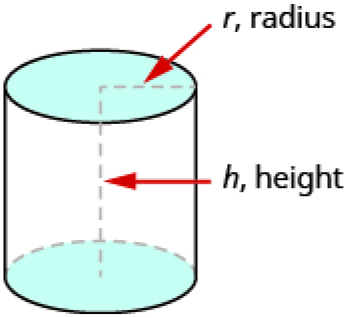
Step 1. **Read** the problem. Draw the figure and	

label it with the given information.	
Step 2. **Identify** what you are looking for.	the volume of the crate
Step 3. **Name.** Choose a variable to represent it.	let V= volume
Step 4. **Translate.** Write the appropriate formula. Substitute.	$V = LWH$ $V = 30 \cdot 25 \cdot 20$
Step 5. **Solve** the equation.	$V = 15{,}000$
Step 6. **Check:** Double check your math.	
Step 7. **Answer** the question.	The volume is 15,000 cubic inches.

Find the Volume of a Cylinder

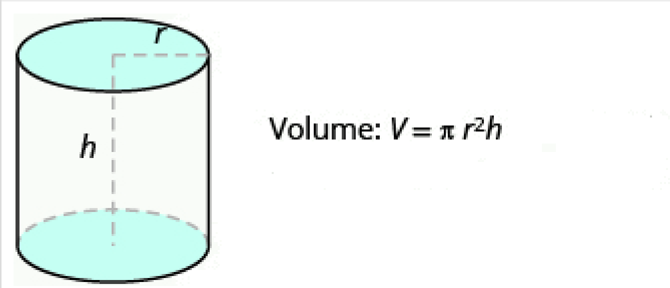
If you have ever seen a can of soda, you know what a cylinder looks like. A **cylinder** is a solid figure with two parallel circles of the same size at the top and bottom. The top and bottom of a cylinder are called the bases. The height h of a cylinder is the distance between the two bases. For all the

cylinders we will work with here, the sides and the height, h , will be perpendicular to the bases.



A cylinder has two circular bases of equal size. The height is the distance between the bases.

Note:
Volume of a Cylinder
For a cylinder with radius r and height h :

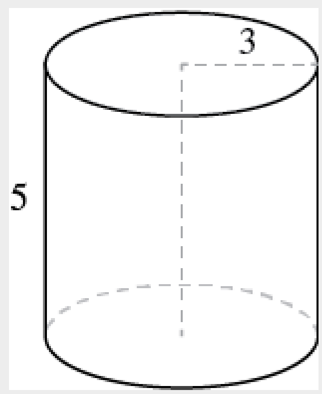


Example:
Exercise:

Problem:

A cylinder has height 5 centimeters and radius 3 centimeters. Find the volume.

Solution:

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the volume of the cylinder
Step 3. **Name.** Choose a variable to represent it.	let V = volume
Step 4. **Translate.** Write the appropriate formula. Substitute. (Use 3.14 for π)	$V = \pi r^2 h$ $V \approx (3.14)3^2 \cdot 5$

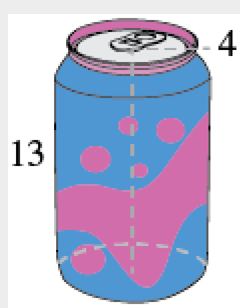
Step 5. **Solve.**	$V \approx 141.3$
Step 6. **Check:** We leave it to you to check your calculations.	
Step 7. **Answer** the question.	The volume is approximately 141.3 cm^3.

Example:
Exercise:

Problem:

Find the volume of a can of soda. The radius of the base is 4 centimeters and the height is 13 centimeters. Assume the can is shaped exactly like a cylinder.

Solution:

Step 1. **Read** the problem. Draw the figure and label it with the given information.	

Step 2. **Identify** what you are looking for.	the volume of the cylinder
Step 3. **Name.** Choose a variable to represent it.	let V = volume
Step 4. **Translate.** Write the appropriate formula. Substitute. (Use 3.14 for π)	$V = \pi r^2 h$ $V \approx (3.14)4^2 \cdot 13$
Step 5. **Solve.**	$V \approx 653.12$
Step 6. **Check:** We leave it to you to check.	
Step 7. **Answer** the question.	The volume is approximately 653.12 cm^3.

Density

What is **density**? Density is a characteristic property of a substance, for liquids and solids. It is a relationship between the mass of the substance and its volume.

Density equals the mass of the substance divided by its volume;
Equation:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

Different substances have different densities. Objects with the same volume but different weight have different densities. The only density you need to remember is for water, as measured in metrics. For every 1 cc or mL of water, it will weigh 1 gram. If we have 1000 mL of water it will weigh 1000 g so 1 L of water weighs 1 kg.

Remember that cc means a box or container that measures one cubic centimeter. This container will hold a milliliter of liquid. If the liquid is water, it will weigh 1 gram. But other liquids have different densities so 1 cc of a liquid that is not water won't necessarily weigh 1 gram.

Since water is used so often you should memorize its density, which is 1 gram/1 cc.

In this section, you will use what you've learned about finding the volume of a rectangular or cylindrical solid to solve density problems.

Recall that mass divided by volume gives us the density of a substance or object.

Equation:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

Knowing two of these three measurements can help us find the third measurement by using the density formula, a proportion, or dimensional analysis.

If we know the mass and volume of an object, we can use the density formula directly and simply divide the mass by the volume to find its density.

Example:
Exercise:

Problem:

A block of an unknown material has a volume of 25 cc and a mass of 42 grams. What is the density of this unknown material?

Solution:

Plug in 25 for the volume and 42 for the mass into the density formula and divide.

Equation:

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{42}{25} = 1.68 \ \text{g/cc}$$

Solving Density Problems Using Proportions

Since density is a rate, we can use proportions to solve the following type of problems:

- What is the mass (given the density and volume)?
- What is the volume (given the density and mass)?

Using the formula for density, we set up a proportion by entering the density on the left side, written as a ratio, and writing the ratio of mass/volume on the right side. Which ever quantity we are trying to find, we will represent with a variable.

Example:
Exercise:

Problem:

Given a density of 3.2 g/cc and a known mass of 8.0 g, find the volume.

Solution:

Let v represent the volume. Then, plugging in the given volume and mass, we obtain the following proportion.
Equation:

$$\frac{3.2}{1} = \frac{8.0}{v}$$

Cross multiplying and solving for v, we get:
Equation:

$$\frac{3.2v}{3.2} = \frac{8.0}{3.2}$$
$$v = 2.5 \text{ cc}$$

Example:
Exercise:

Problem:

What is the mass of a block measuring a volume of 2.8 cc and density of 2.54 g/cc?

Solution:

Let m represent the mass. Then, setting up a proportion and solving, we have
Equation:

$$\frac{2.54}{1} = \frac{m}{2.8}$$

$$m = (2.54)(2.8)$$
$$m = 7.112 \text{ g}$$

Example:
Exercise:

Given a piece of wood and its density, find its mass.

Problem:

If the density of oak wood is 0.85 g/cc, what is the mass of a piece of wood that is 20 cm by 30 cm by 2 cm? Give the answer in both kilograms and in pounds.

Solution:

Step 1: Find the volume. This is a rectangular solid, so the volume can be found by multiplying the three given dimensions together.

Volume = (20 cm)(30 cm)(2 cm) = 1200 cm^3 (cc)

Step 2: Find the mass by using its volume and density. We will use a

proportion for this example. The left side will be the density, written as a ratio, and the right side will be the mass/volume. Let m = mass.

Equation:

$$\frac{0.85 \text{ g}}{1 \text{ cc}} = \frac{m}{1200 \text{ cc}}$$

$$m = (0.85)(1200)$$
$$m = 1020 \text{ g}$$

Now, we need to convert the mass into kg, which we can do by moving the decimal three places to the left. Thus, the mass is 1.02 kg.

Step 3: Lastly, we can use dimensional analysis to convert 1.02 kg into pounds:

Equation:

$$\frac{1.02 \ \cancel{\text{kg}}}{1} \cdot \frac{2.2 \text{ lb.}}{1 \ \cancel{\text{kg}}} = 2.244 \text{ lb.}$$

Example:

Exercise:

Problem:

A cylinder has a radius of 5 mm and a height of 12 mm. If the density of the material the cylinder is made of is 1.64 g/cc, what is the weight of the cylinder? Find the weight in both grams and pounds.

Solution:

Step 1: Find the volume. This is a cylinder, so we need to use formula $V = \pi r^2 h$. However, to use the density formula, we need the volume to be in cc (cm^3). So, first we need to convert the radius and height to cm by moving each decimal one place to the left. This gives us a radius of 0.5 cm and a height of 1.2 cm. We will use 3.14 for π

V = (3.14)(0.5 cm)^2(1.2 cm) = 0.942 cc.

Step 2: Use the volume and density to find the mass. We'll use a proportion. We start by writing the left side as the density, written as a ratio, and the right side as mass/volume. Let m = mass.

Equation:

$$\frac{1.64 \text{ g}}{1 \text{ cc}} = \frac{m}{0.942 \text{ cc}}$$

$$m = (1.64)(0.942)$$
$$m = 1.54 \text{ g}$$

Step 3: Convert 1.54 grams into pounds. For this conversion, we use a dimensional analysis:

Equation:

$$\frac{1.54 \cancel{\text{g}}}{1} \cdot \frac{1 \cancel{\text{kg}}}{1000 \cancel{\text{g}}} \cdot \frac{2.2 \text{ lb}}{1 \cancel{\text{kg}}} \approx 0.0034 \text{ lb.}$$

Solving Density Problems Using Dimensional Analysis

We can also solve the following type of density problems using dimensional analysis:

- Find the mass of an object, given its density and volume.
- Find the volume of an object, given its density and mass.

One benefit of using dimensional analysis to solve density problems is that sometimes we want to find the mass in units other than grams or the volume in units other than mL or cc. Since density is usually given in g/mL or g/cc, using dimensional analysis allows us to not only find the mass or volume, but also convert into different units, all in the same process. It's also useful when the volume or mass that is given is not in mL or g respectively.

Example:
Exercise:

Problem:

What is the mass of a block measuring a volume of 2.8 cc and density of 2.54 g/cc?

Solution:

To use dimensional analysis for this type of problem, you would start with the given volume, written as a ratio, and then use the density as a unit ratio, writing it in such a way that the correct units will cross cancel.

$$\frac{2.8\ \cancel{cc}}{1} \cdot \frac{2.54\ g}{1\ \cancel{cc}} = 7.112\ g$$

Example:
Exercise:

Problem:

Given a density of 3.2 g/cc and a known mass of 8.0 g, find the volume.

Solution:

To use dimensional analysis for this type of problem, you would start with the given mass, written as a ratio, and then use the density as a unit ratio, writing it in such a way that the correct units will cross cancel.

$$\frac{8.0\ \cancel{g}}{1} \cdot \frac{1\ cc}{3.2\ \cancel{g}} = 2.5\ cc$$

Example:
Exercise:

Problem:

Given a density of gold as 19.3 g/cc, find the mass of 200 cc of gold in **kilograms**.

Solution:

$$\frac{200\ \cancel{cc}}{1} \cdot \frac{19.3\ \cancel{g}}{1\ \cancel{cc}} \cdot \frac{1\ kg}{1000\ \cancel{g}} = 3.86\ kg$$

Example:
Exercise:

Problem:

If the density of gasoline is 0.675 g/cc find the mass of a gallon of gasoline.

Solution:

Since we are given the weight of the gasoline, which is in gallons, rather than its mass in grams, we start by converting the gallons into grams, and then we can use the density rate to finish the conversion.

$$\frac{1\ \cancel{\text{gal.}}}{1} \cdot \frac{4\ \cancel{\text{qt.}}}{1\ \cancel{\text{gal.}}} \cdot \frac{946\ \cancel{\text{mL}}}{1\ \cancel{\text{qt.}}} \cdot \frac{0.675\ \text{g}}{1\ \cancel{\text{mL}}} = 2{,}554.2\ \text{g}\ (\text{or}\ \approx 2.55\ \text{kg})$$

Note:
Weight versus Mass
The terms "weight" and "mass" are often used interchangeably. However, they actually do not measure the same thing. Watch this video to learn more about weight versus mass.

Homework

Find Volume of Rectangular Solids

In the following exercises, find the volume of the rectangular solid with the given dimensions.
Exercise:

Problem: length 8 feet, width 9 feet, and height 11 feet.

Solution:

792 cu. ft

Exercise:

Problem: length 15 feet, width 12 feet, and height 8 feet.

Exercise:

Problem: length 2 meters, width 1.5 meters, height 3 meters

Solution:

9 m^3

Exercise:

Problem: length 5 feet, width 8 feet, height 2.5 feet

Exercise:

Problem: length 3.5 yards, width 2.1 yards, height 2.4 yards

Solution:

17.64 cu. yd.

Exercise:

Problem:

length 8.8 centimeters, width 6.5 centimeters, height 4.2 centimeters

Exercise:

Problem:

Moving van A rectangular moving van has length 16 feet, width 8 feet, and height 8 feet.

Solution:

1,024 cu. ft

Exercise:

Problem:

Gift box A rectangular gift box has length 26 inches, width 16 inches, and height 4 inches.

Exercise:

Problem:

Carton A rectangular carton has length 21.3 cm, width 24.2 cm, and height 6.5 cm.

Solution:

3,350.49 cu. cm

Exercise:

Problem:

Shipping container A rectangular shipping container has length 22.8 feet, width 8.5 feet, and height 8.2 feet.

Find the Volume of a Cylinder

In the following exercises, find the volume of the cylinder with the given radius and height. Use 3.14 for π. Round answers to the nearest hundredth.

Exercise:

Problem:

Find the volume of the cylinder with given radius 2 ft and height 8 ft.

Solution:

100.48 ft^3

Exercise:

Problem:

Find the volume of a can of paint with radius 8 centimeters and height 19 centimeters. Assume the can is shaped exactly like a cylinder.

Exercise:

Problem:

Find the volume of the cylinder with radius 4 cm and height 7cm.

Solution:

351.68 cu. cm

Exercise:

Problem:

Find the volume of a cylindrical drum with radius 2.7 feet and height 4 feet. Assume the drum is shaped exactly like a cylinder.

Exercise:

Problem: radius 3 feet, height 9 feet

Solution:

254.34 cu. ft

Exercise:

Problem: radius 5 centimeters, height 15 centimeters

Exercise:

Problem: radius 1.5 meters, height 4.2 meters

Solution:

29.67 cu. m

Exercise:

Problem: radius 1.3 yards, height 2.8 yards

Exercise:

Problem:

Coffee can A can of coffee has a radius of 5 cm and a height of 13 cm. Find its volume.

Solution:

1,020.5 cu. cm

Exercise:

Problem:

Snack pack A snack pack of cookies is shaped like a cylinder with radius 4 cm and height 3 cm. Find its volume.

Exercise:

Problem:

Barber shop pole A cylindrical barber shop pole has a diameter of 6 inches and height of 24 inches. Find its volume.

Solution:

678.24 in^3

Exercise:

Problem:

Architecture A cylindrical column has a diameter of 8 feet and a height of 28 feet. Find its volume.

Density Problems

Exercise:

Problem:

A cylinder has a radius of 75 mm and a height of 135 mm. If the density of the material the cylinder is made of is 0.84 g/cc, what is the mass of the cylinder? Find the mass in grams, **using a proportion** and then convert to pounds.

Solution:

2003 g and 4.4 lb.

Exercise:

Problem:

A rectangular block is made of material with a density of 2.8 g/cc. If the dimensions of the block are 35 cm, 52 cm, and 4.2 cm, what is its mass in kilograms? What is its weight in pounds? **Use dimensional analysis** to find both answers.

For the next four exercises, **use a proportion** to solve the problem.

Exercise:

Problem:

Given a volume of 32 cc of mercury and given the density of mercury to be 13.6 g/cc, find the weight.

Solution:

435.2 g

Exercise:

Problem:

Given the density is 3.72 g/cc and the volume is 15 cc, find the mass.

Exercise:

Problem:

Given a density of 0.75 g/cc and a known mass of 1.2 g, find the volume.

Solution:

1.6 cc

Exercise:

Problem:

If the density of glass is 2.2 g/cc at room temperature find the volume of a piece of glass weighing 1500 g.

For the following exercises, **use dimensional analysis** to find the answer.

Exercise:

Problem:

What is the volume of 15 kg of water? Use both mL and liters. (Remember that water has a density of 1 g/cc or 1 g/mL.)

Solution:

15,000 mL = 15 L

Exercise:

Problem: Find the volume of a 45 g object with a density of 2.75 g/cc.

Exercise:

Problem:

What is the mass of an object measuring a volume of 56 cc and having a density of 1.12 g/cc?

Solution:

62.72 g

Exercise:

Problem:

What is the volume of an object weighing 125 grams and having a density of 4.8 g/cc?

Converting Rates

Learning Objectives

By the end of this lesson, you will be able to use dimensional analysis to:

- Convert rates.
- Convert measurements with squared or cubed units.
- Convert mi./hr. to m/min. for metabolic calculations.
- Convert metric rates.

Converting Rates

Rates have 2 labels that need to be changed in the one process. **Don't stop with a partial set up. Convert both labels in one process.**

Example:
Exercise:

Problem: Convert 20.5 m/hr. to ft./min.

Solution:

$$\frac{20.5\ \cancel{\text{m}}}{1\ \cancel{\text{hr.}}}\cdot\frac{100\ \cancel{\text{cm}}}{1\ \cancel{\text{m}}}\cdot\frac{1\ \cancel{\text{in.}}}{2.54\ \cancel{\text{cm}}}\cdot\frac{1\text{ ft.}}{12\ \cancel{\text{in.}}}\cdot\frac{1\ \cancel{\text{hr.}}}{60\text{ min.}}=\frac{(20.5)(100)\text{ ft.}}{(2.54)(12)(60)\text{min.}}$$

$$=\frac{2050\text{ ft.}}{1828.8\text{ min.}}$$

$$\approx 1.12\text{ ft./min.}$$

When you are converting rates, concentrate on converting one label

(unit) at a time. So, the first 3 unit ratios are used to convert the m to ft. and the last unit ratio is used to convert the hr. to min, using the fact that there are 60 minutes in 1 hour. Note that the first “hr.” is in the denominator, so the unit ratio must have “hr.” in the numerator in order to cancel each other.

Notice how, after all the unit ratios are given so that we are left with ft. in the numerator and min. in the denominator, the ratios are then multiplied by first multiplying all the numerators together (ignoring the 1’s) and then multiplying the denominators together (again, ignoring the 1’s). Units that have not been canceled are included in this multiplication. The last step is to divide and label the answer correctly.

Conversions with Squared or Cubed Units

When converting squared or cubed units, keep in mind what “squared” or “cubed” means. For example, square feet means ft.^{2}, which means (ft.)(ft.). So, to convert ft.^{2} into in.^{2}, we would need two unit ratios, one for each “ft.”.

Example:
Exercise:

Problem: Convert 40,000 sq. cm to sq. ft.

Solution:

$$\frac{40{,}000\ \cancel{\text{cm}^2}}{1}\cdot\frac{1\ \cancel{\text{in.}}}{2.54\ \cancel{\text{cm}}}\cdot\frac{1\ \cancel{\text{in.}}}{2.54\ \cancel{\text{cm}}}\cdot\frac{1\ \text{ft.}}{12\ \cancel{\text{in.}}}\cdot\frac{1\ \text{ft.}}{12\ \cancel{\text{in.}}}=\frac{40{,}000\ \text{ft.}^2}{(2.54)^2(12)^2}=\frac{40{,}000\ \text{ft.}^2}{929.0304}\approx 43\ \text{ft.}^2$$

Notice that we need two unit ratios relating cm and in. so that both cm's in the cm^2 label will cancel. Then, we need two unit ratios relating in. and ft. to cancel the two "in." labels and end up with two "ft." labels, which multiplied together results in the "$ft.^2$" label, which we were converting to.

Here is another possible set up:

$$\frac{40{,}000\ \cancel{cm^2}}{1}\cdot\left(\frac{1\ \cancel{in.}}{2.54\ \cancel{cm}}\right)^2\cdot\left(\frac{1\ ft.}{12\ \cancel{in.}}\right)^2=\frac{40{,}000\ ft.^2}{(2.54)^2(12)^2}=\frac{40{,}000\ ft.^2}{929.0304}\approx 43\ ft.^2$$

Caution: When using this type of set-up, you must be careful to remember that everything inside the parentheses gets squared – the units **and** the numbers.

Metabolic Calculations

Here are a few basics that will be seen for Metabolic Calculations, like those shown in Evaluating Formulas. These types of equations calculate oxygen consumption and energy expenditure for a given type of exercise.

It is common to change miles per hour to meters per min when using formulas for Metabolic Calculations. Because it is so common, there is a standardized number to use, which is 26.8 m/min. Let's see where this standardized number comes from. Start with a rate of 1 mile per hour.

Example:
Exercise:

Problem: Convert 1 mi./hr. to m/min.

Solution:

$$\frac{1\ \cancel{\text{mi.}}}{1\ \cancel{\text{hr.}}} \cdot \frac{5280\ \cancel{\text{ft.}}}{1\ \cancel{\text{mi.}}} \cdot \frac{12\ \cancel{\text{in.}}}{1\ \cancel{\text{ft.}}} \cdot \frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in.}}} \cdot \frac{1\text{ m}}{100\ \cancel{\text{cm}}} \cdot \frac{1\ \cancel{\text{hr.}}}{60\text{ min.}} \approx 26.8\text{ m/min.}$$

Notice that all the same unit ratios would be used with any number of miles. So, if we need to convert any number of miles/hour to m/min, we can simply multiply that number times 26.8. We only use this shortcut because converting from mi./hr. to m/min. is very common in exercise movement science.

Example:
Exercise:

Problem:

Convert 2.34 miles per hour to meters per minute by multiplying it by the standard 26.8 and then verify with dimensional analysis.

Solution:

$$2.34 \times 26.8 \approx 62.71\text{ m/min.}$$

Let’s check this answer with dimensional analysis.

$$\frac{2.34\ \cancel{\text{mi.}}}{1\ \cancel{\text{hr.}}} \cdot \frac{5280\ \cancel{\text{ft.}}}{1\ \cancel{\text{mi.}}} \cdot \frac{12\ \cancel{\text{in.}}}{1\ \cancel{\text{ft.}}} \cdot \frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in.}}} \cdot \frac{1\text{ m}}{100\ \cancel{\text{cm}}} \cdot \frac{1\ \cancel{\text{hr.}}}{60\text{ min.}} \approx 62.76\text{ m/min.}$$

There is a slight difference in answers due to rounding, but it is standard to just use 26.8 when converting miles per hour to meters per minute for metabolic calculations.

Metric Rates

More Metric Prefixes
Previously, we discussed the fact that for the metric system, there are basic units for the categories of linear, volume, and mass, which are meter, liter, and gram, respectively. We also learned the prefixes of kilo, hecto, deka, deci, centi, and milli. We will now add four more prefixes and their abbreviations, which are commonly used in the health and physical sciences:

- mega (M) for one million times a standard unit.
- micro (mc) for one millionth of a standard unit.
- nano (use a lower case n) for one billionth of a standard unit.
- pico (p) for one trillionth of a standard unit.

You should recognize these prefixes and their abbreviations. Although, the metric prefix chart will be provided whenever needed.

We now have an extended metric prefix chart that we can use for converting between metric units by shifting the decimal, which is an extension of the chart used in the Metric System.

Mega						base						micro			nano			pico
M			k	h	da	g m L	d	c	m			mc			n			p

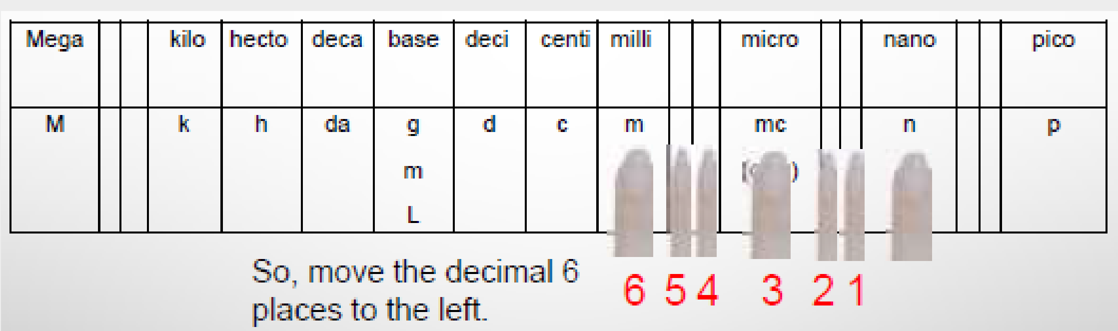
Note the gaps in the chart. These represent powers of 10 that we won't be using the prefixes for, but they must be used when shifting the decimal to convert between metric units.

Example:
Exercise:

Problem: Convert 52,375 nanograms to milligrams.

Solution:

Start in the chart at nano and count how many places and what direction you need to move in the chart to end at milli, making sure you count the gaps as well.



Mega			kilo	hecto	deca	base	deci	centi	milli			micro			nano			pico
M			k	h	da	g m L	d	c	m			mc			n			p

Therefore, 52,375 ng = 0.052375 mg.

When both labels of a rate are metric measurements, we can convert to another metric rate by either shifting the decimal point or with dimensional analysis. Be sure to watch for the denominator that is wanted. If it says per mL that means one mL.

Example:
Exercise:

Problem:

Convert 180 mg per mL to g per dL, first using dimensional analysis, and then by shifting the decimal.

Solution:

Using dimensional analysis:

$$\frac{180\ \cancel{mg}}{1\ \cancel{mL}} \cdot \frac{1\ g}{1000\ \cancel{mg}} \cdot \frac{1000\ \cancel{mL}}{1\ \cancel{L}} \cdot \frac{1\ \cancel{L}}{10\ dL} = 18\ g/dL$$

In contrast, to convert this rate by shifting the decimal, we move the decimal on the numerator (180 mg) three places to the left, to convert it to grams, and then move the decimal on the denominator (1 mL) two places to the left, to convert it to deciliters. The last step is to reduce the denominator to 1 dL by dividing the numerator by the denominator.

$$\frac{180\ \text{mg}}{1\ \text{mL}}\begin{matrix}\rightarrow\\\rightarrow\end{matrix}=\frac{0.18\ \text{g}}{0.01\ \text{dL}}=18\ \text{g/dL}$$

Example:
Exercise:

Problem:

Convert 2.5 g/L to mg per 100 mL, first using dimensional analysis, and then by shifting the decimal.

Solution:

Using dimensional analysis:

$$\frac{2.5\ \cancel{\text{g}}}{1\ \cancel{\text{L}}}\cdot\frac{1000\ \text{mg}}{1\ \cancel{\text{g}}}\cdot\frac{1\ \cancel{\text{L}}}{1000\ \cancel{\text{mL}}}=\frac{2.5\ \text{mg}}{1\ \text{mL}}\cdot\frac{100}{100}=\frac{250\ \text{mg}}{100\ \text{mL}}$$

Notice that we first convert to mg/mL and then we multiply both the numerator and denominator by 100 to obtain a denominator of 100 mL.

By shifting the decimal:

$$\frac{2.5\ \text{g}}{1\ \text{L}}\begin{matrix}\rightarrow\\\rightarrow\end{matrix}=\frac{2500\ \text{mg}}{1000\ \text{mL}}=\frac{2.5\ \text{mg}}{1\ \text{mL}}\cdot\frac{100}{100}=\frac{250\ \text{mg}}{100\ \text{mL}}$$

Notice that the last step is the same as with dimensional analysis. However, we could also have divided both the numerator and denominator by 10, instead of reducing and then multiplying, since 1000 divided by 10 is also 100.

Homework

For the following exercises, use dimensional analysis to make the conversions. Be sure to include labels on your answers.

Exercise:

Problem: Convert 343 miles in 3.5 hr. to kilometers per 1 hr.

Solution:

158.06 km/hr.

Exercise:

Problem: Convert 10 mg per lb. to g per kg

Exercise:

Problem: Convert 0.85 g per liter to mg per ml

Solution:

0.85 mg/mL

Exercise:

Problem:

If a snail travels 12 feet over night in 8 hours, how many inches per second does it travel?

Exercise:

Problem: Convert 16.5 square feet to square yard.

Solution:

1.83 yd.^{2}

Exercise:

Problem: Convert 1.45 sq. meters to sq. inches.

Exercise:

Problem: Change 0.55 sq. m per hour to sq. ft per minute.

Solution:

0.099 ft.^{2}/min.

Exercise:

Problem: Convert 2750 sq. cm to sq feet.

Exercise:

Problem:

Convert 5 miles per hour to meters per min., using the standard conversion factor of 26.8 and then verify with dimensional analysis.

Solution:

Calculate: 5 (26.8) = 134 meters per minute

Verify: $\frac{5\ \cancel{\text{mi.}}}{1\ \cancel{\text{hr.}}} \cdot \frac{5280\ \cancel{\text{ft.}}}{1\ \cancel{\text{mi.}}} \cdot \frac{12\ \cancel{\text{in.}}}{1\ \cancel{\text{ft.}}} \cdot \frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in.}}} \cdot \frac{1\text{ m}}{100\ \cancel{\text{cm}}} \cdot \frac{1\ \cancel{\text{hr.}}}{60\text{ min.}} = 134.112\text{ m/min.}$

Exercise:

Problem: Change 0.05 g per liter to mcg per mL.

Exercise:

Problem: Change 2500 mg per liter to g per 100 mL.

Solution:

0.25g/100mL

Exercise:

Problem: Change 1.06 mg per liter to mcg per dL.

Exercise:

Problem: Change 0.2305 g per liter to mg per mL.

Solution:

0.2305 mg/mL.

Exercise:

Problem: Change 6.2 kg per liter to g per mL.

Exercise:

Problem:

A current reading for driving under the influence of marijuana is a reading of THC blood content. If the reading is higher than 5 nanograms per milliliter, then the person legally can’t be driving. Change this reading to mg per dl.

Solution:

0.0005 mg/dL

Exercise:

Problem:

A blood report shows a reading of 0.006 g per liter, convert to mcg per dl for this blood report.

Exercise:

Problem: Given a reading of 0.3 mg per liter change to mcg per dl.

Solution:

30mcg/dL

Exercise:

Problem: Convert 21.2 miles per gallon to feet per cup.

Dosage

Learning Objectives

By the end of this lesson, you will be able to:

- Use the correct routine for dosage conversions.
- Use dimensional analysis to calculate dosages from doctor's order.
- Calculate dosages based on body size.
- Calculate dosages that include reconstituting a powdered drug to a liquid.

Recall the following abbreviations from Medical Abbreviations and Systems:

- I.M. (intramuscular) This is a small amount to be injected with a syringe.
- I.V. (intravenously) This is a larger amount to be infused with an IV bag.
- P.O. (by mouth) This is to be swallowed.
- S.C. (subcutaneously which is under the skin) This is a small amount for a syringe.

The useable form of drug for the patient depends on what supply is on hand. Some medicines come in different strengths; a tablet could be 125 mg or 500 mg.

The doctor’s order could also be written as drug per kilogram or drug per square meter. In this case the patient’s size will need to be a factor in the conversion process. For example, a doctor’s order may be written as 0.25 mg of drug per 1 sq. meter. The size of the patient will then determine how many mg are to be given.

Dimensional analysis can be used to change the order to the amount to be administered, but note that the label of square meter is eliminated by the body size and thus not converted as a squared unit.

There is a preferred routine to set up the dosage conversion, starting first with the doctor’s order.

Here is the plan: go from order to size to supply.

1. Order
2. Patient size (this may or may not be part of the order)
3. Supply (this is the type of medicine with its concentration on the label; a different bottle may have a different concentration.)

Example:
Exercise:

Problem:

A patient’s BSA is 0.72 sq. m. Estimate how much of the drug will be administered based on the order of 0.25 mg per sq. meter. The supply on hand has a concentration of 1 mg/10 mL.

Solution:

$$\frac{0.25\ \cancel{\text{mg}}}{1\ \cancel{\text{sq. m}}}\cdot\frac{0.72\ \cancel{\text{sq. m}}}{1}\cdot\frac{10\ \text{mL}}{1\ \cancel{\text{mg}}}=1.8\ \text{mL}$$

Notice how we start by writing the order as a ratio. Then, we multiply by the patient's size. Lastly, we multiply by the concentration of the supply, written as a ratio with mg in the denominator so the mg will cross cancel.

Note:
Try It
Exercise:

Problem:

The patient has a BSA of 1.8 sq. m. Estimate how much of the drug will be administered based on the order of 0.25 mg per sq. m. The concentration of the supply is 1 mg/10 mL.

Solution:

$$\frac{0.25\ \cancel{\text{mg}}}{1\ \cancel{\text{sq. m}}} \cdot \frac{1.8\ \cancel{\text{sq. m}}}{1} \cdot \frac{10\ \text{mL}}{1\ \cancel{\text{mg}}} = 4.5\ \text{mL}$$

Example:
Exercise:

Problem:

The order is for 0.05 mg per kg and the patient weighs 92.6 kg. Determine how much drug in mL will be administered. The concentration of the supply is 10 mg/22 mL.

Solution:

$$\frac{0.05\ \cancel{\text{mg}}}{1\ \cancel{\text{kg}}} \cdot \frac{92.6\ \cancel{\text{kg}}}{1} \cdot \frac{22\ \text{mL}}{10\ \cancel{\text{mg}}} = 10.2\ \text{mL}$$

Example:
Exercise:

Problem:

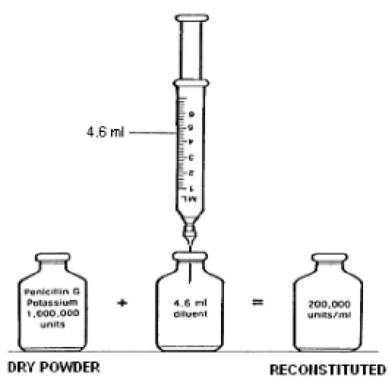
A physician has ordered mephenytoin (Mesatoin) 150 mg/m^2 tid P.O. for control of grand mal seizures. The BSA of the patient is 1.1 m^2. Using a supply concentration of 1 capsule/82.5 mg, how many capsules are needed for the day?

Solution:

Recall that tid means three times per day. We first use dimensional analysis to calculate a single dose.

$$\frac{150\ \cancel{mg}}{1\ \cancel{m^2}} \cdot \frac{1.1\ \cancel{m^2}}{1} \cdot \frac{1\ \text{cap}}{82.5\ \cancel{mg}} = 2\ \text{caps}$$

Since a single dose is 2 capsules and they are to take three doses per day, that means they will need to take 6 capsules per day.



The diagram above shows reconstituting a

powdered drug into a liguid, to be administered with a syringe. Some vials contain a specific amount of powdered drug measured in units or grams or milligrams. This powder must be changed into a liquid by adding a specified amount of diluent, such as sterile water.

This vial of dry powder contains 1,000,000 units of the drug, which is a type of penicillin. The directions on the label of the dry powder will include how to mix the drug by stating how much liquid to add (4.6 mL in this example). This is not part of the dosage calculation. After following the mixing directions, the concentration of the reconstituted solution to be administered will be 200,000 units per mL, as shown on its label. This is considered the supply.

Example:
Exercise:

Problem:

Here is an example where the supply is a powdered drug in a vial. Find a single dose in milliliters.

- Doctor’s order: 50,000 units I.M.
- Supply: 100,000 units/mL. (After reconstituting, the strength of the solution in the vial is 100,000 units per mL.)

Solution:

$$\frac{50{,}000\ \cancel{\text{units}}}{1} \cdot \frac{1\text{ mL}}{100{,}000\ \cancel{\text{units}}} = 0.5\text{ mL}$$

Notice that we start with the doctor's order. Then, we multiply the unit ratio formed by the supply, which is the strength of the solution indicated on the label, placing the units in the denominator so the units will cross cancel.

Note:
Try It
Exercise:

Problem:

The order is 150,000 units to be administered from a vial containing 1,000,000 units, to be reconstituted with 5 mL of diluent. The supply concentration of the reconstituted vial is 200,000 units per cc. How many cc should be withdrawn from the vial?

Solution:

$$\frac{150{,}000\ \cancel{\text{units}}}{1} \cdot \frac{1\text{ cc}}{200{,}000\ \cancel{\text{units}}} = 0.75\text{ cc}$$

Homework

In the following exercises, use dimensional analysis to determine a single dose. If the drug is to be administered more than once, then multiply the single dose by the total number of doses that will be given.

Exercise:

Problem:

The doctor’s order is for 0.4 mcg per sq. meter. If the patient has a BSA of 0.3 sq. meters and the supply concentration is 1 mcg/10 mL, how much drug will be administered?

Solution:

1.2 mL

Exercise:

Problem:

The order is cycloserine (Seromycin) 0.25 g P.O. q8h for 2 weeks. Each capsule of the antitubercular drug contains 250 mg. How many capsules will you need for two weeks?

Exercise:

Problem:

You have asked the patient for the name of his antihypertensive drug. The drug is Minipress 0.002 g/cap. The label on the bottle reads: 1 capsule tid. How many milligrams of drug is contained in one capsule? How many milligrams are to be administered each day?

Solution:

6 mg/day

Exercise:

Problem:

The antianxiety drug prazepan has been prescribed for the patient, 20 mg tid. The medication is available in 5 mg capsules. How many capsules are prescribed per day?

Exercise:

Problem:

A patient has an order for the antidepressant drug, Zoloft, 0.05 g P.O. daily. Each scored tablet is labeled 100 mg. How many tablets will you administer? (A scored tablet has a dividing mark across the tablet so it can be cut in half.)

Solution:

0.5 tab

Exercise:

Problem:

A patient has been having difficulty swallowing and the physician changed Compazine from tablets tosuppositories. The order is Compazine suppositories (rectal) q6h prn 10 mg. If each suppository is 5 mg, how many suppositories would you administer for the day? (prn is the abbreviation for "as needed.")

Exercise:

Problem:

The order is 2.5 milligrams per kilogram of a drug for a patient weighing 68 kilograms. Each capsule contains 50 milligrams. How many capsules would you administer to the patient?

Solution:

= 3.4 cap , so give 3 capsules

Exercise:

Problem:

Cromolyn (Opticrom) nasal solution has been prescribed for its antiasthmatic action. The solution is labeled 5.2 mg/metered spray. If the patient administers 3 metered sprays via an inhaler, how many milligrams are inhaled?

Exercise:

Problem:

Your patient must receive 0.5 mg of digoxin P.O. stat. Tablets available contain 0.125 mg. How many tablets will you administer to the patient?

Solution:

4 tabs

Exercise:

Problem:

The prescriber has written an order for 3 mg hydromorphone (Dilaudid) an opiod P.O., q4h prn. Label reads 10 mg = 15 mL. How many milliliters will you give your patient?

Exercise:

Problem:

How many tablets are needed over 24 hours if the order is 300 mg every 4 hours and the supply is 150 mg/tab?

Solution:

12 tablets

Exercise:

Problem:

The order is 100 mg b.i.d and there is 25 mg/cap available. How many caplets will be given to the patient over 24 hours?

Exercise:

Problem:

The doctor orders 15 mg/kg of a drug. The patient weighs 66 kg The drug is supplied as 100 mg/2mL. How many mL will you administer per dose?

Solution:

19.8 mL

Exercise:

Problem:

The order is for Ibuprofen 6 mg/kg by mouth every 4 hours as needed for pain for a child. The child weighs 72 lbs. The pharmacy dispenses the Ibuprofen at 25 mg/mL. How many mL will you administer in 24 hours?

Molarity

Learning Objectives

By the end of this lesson, you will be able to:

- Understand the meaning of molarity.
- Calculate solution concentrations, in terms of molarity, using dimensional analysis.

Molarity of Solutions

A **mole** is a unit that measures 6.02×10^{23} particles of an element. This is a very large count! The molecules are weighed though and not counted. Mole is abbreviated as mol. The weight of a mole varies depending on the density of the element. There is a known weight for each element or compound and it is called the atomic weight. This atomic weight is found on the periodic chart of elements that you might use in your science classes.

Molarity is a rate or comparison of “moles per liter.” It shows a concentration of a solution as the ratio of moles of solute to liters of solution.

Molarity is the rate of moles per liter. Molarity is abbreviated as M. (Note: If you see a capital M without a base it is Molarity; a capital M with a base unit is the metric prefix of Mega.)

Equation:

$$M = \frac{\text{mol solute}}{\text{L solution}}$$

Example:

Exercise:

Problem:

A 355-mL soft drink sample contains 0.133 mol of sucrose (table sugar). What is the molarity of sucrose in the beverage?

Solution:

Since molarity, by definition, is a rate, we can use dimensional analysis to calculate molarity. We start by combining the given volume and moles into a single ratio, being sure to place moles in the numerator and mL in the denominator. Then, we use dimensional analysis to convert the mL to liters.

$$\frac{0.133\text{ mol}}{355\ \cancel{\text{mL}}} \cdot \frac{1000\ \cancel{\text{mL}}}{1\text{ L}} \approx 0.375\text{ mol/L} = 0.375\text{ M}$$

Example:
Exercise:

Problem:

Find the molarity of 1.462 g of NaCl in 250 mL of solution, given the atomic weight of NaCl is 58 g/mol.

Solution:

For this example, not only do we need to convert the mL to liters, but we also need to use the given atomic weight to convert the grams to moles.

$$\frac{1.462\ \cancel{\text{g}}}{250\ \cancel{\text{mL}}} \cdot \frac{1\text{ mol}}{58\ \cancel{\text{g}}} \cdot \frac{1000\ \cancel{\text{mL}}}{1\text{ L}} \approx 0.1008\text{ mol/L} = 0.1008\text{ M}$$

Example:
Exercise:

Problem:

Distilled white vinegar is a solution of acetic acid in water. A 0.500-L vinegar solution contains 25.2 g of acetic acid. What is the concentration of the acetic acid solution in units of molarity, given that the atomic weight of acetic acid is 60.052 g/mol?

Solution:

As in the previous example, we need to use the given atomic weight to convert the grams to moles.

$$\frac{25.2\ \cancel{g}}{0.5\ L} \cdot \frac{1\ mol}{60.052\ \cancel{g}} \approx 0.8393\ mol/L = 0.8393\ M$$

Example:
Exercise:

Problem:

If we mix a packet of sugar into one cup of coffee, what is the molarity of this solution? One packet of sugar weighs 2.8 grams. A cup of coffee is about 250 mL. As an estimate, there are about 342.3 g per mol for sugar.

Solution:

$$\frac{2.8\ \cancel{g}}{250\ \cancel{mL}} \cdot \frac{1\ mol}{342.3\ \cancel{g}} \cdot \frac{1000\ \cancel{mL}}{1\ L} \approx 0.0327 mol/L = 0.0327\ M$$

Homework

For the following problems, use dimensional analysis to find the molarity.

Exercise:

Problem:

Find the molarity of 6.5 g of methanol in 500 ml of solution, given the atomic weight of methanol is 32.04 g/mol.

Solution:

0.406 M

Exercise:

Problem:

Find the molarity of 1.50 grams of NaCl in 500 ml of solution, given the atomic weight of NaCl is 58 g/mol.

Exercise:

Problem:

Find the molarity of 12.08 g of a substance in 900 mL of solution if one mole of the substance weighs 58.44 grams.

Solution:

0.2297 M

Exercise:

Problem:

Find the molarity of 2.05 mg of lactose in 300 ml of solution, given the atomic weight of lactose is 342.3 g/mol.

Exercise:

Problem:

Find the molarity of 5.3 grams of sodium carbonate in 750 ml of solution, given the atomic weight of sodium carbonate is 106 g/mol.

Solution:

0.0667 M

Exercise:

Problem:

Find the molarity of 0.645 g of NaCl (given 1 mol = 58 g) in 750 mL of water.

Exercise:

Problem:

Find the molarity of 15 grams of NaCl (given 1 mol = 58 g) in 2 L of solution.

Solution:

0.129 M

Exercise:

Problem:

Find the molarity of 50 mg of a substance in 800 mL of water if one mole of the substance weighs 49 grams.

Scientific Notation

Learning Objectives

By the end of this lesson, you will be able to

- Read and understand scientific notation.
- Convert between decimal notation and scientific notation.
- Use scientific notation on a calculator.

Convert from Decimal Notation to Scientific Notation

Remember working with place value for whole numbers and decimals? Our number system is based on powers of 10. We use tens, hundreds, thousands, and so on. Our decimal numbers are also based on powers of tens—tenths, hundredths, thousandths, and so on.

Consider the numbers 4000 and 0.004. We know that 4000 means 4×1000 and 0.004 means $4 \times \frac{1}{1000}$. If we write the 1000 as a power of ten in exponential form, we can rewrite these numbers in this way:
Equation:

$$\begin{array}{ll} 4000 & 0.004 \\ 4 \times 1000 & 4 \times \frac{1}{1000} \\ 4 \times 10^{3} & 4 \times \frac{1}{10^{3}} \\ & 4 \times 10^{-3} \end{array}$$

When a number is written as a product of two numbers, where the first factor is a number greater than or equal to one but less than 10, and the second factor is a power of 10 written in exponential form, it is said to be in *scientific notation.*

Note:

Scientific Notation

A number is expressed in **scientific notation** when it is of the form

Equation:

$$a \times 10^{n}$$

where $a \geq 1$ and $a < 10$ and n is an integer.

It is customary in scientific notation to use $\times$ as the multiplication sign, even though we avoid using this sign elsewhere in algebra.

Scientific notation is a useful way of writing very large or very small numbers. It is used often in the sciences to make calculations easier.

If we look at what happened to the decimal point, we can see a method to easily convert from decimal notation to scientific notation.

$4000. = 4 \times 10^{3}$	$0.004 = 4 \times 10^{-3}$
$4000. = 4 \times 10^{3}$	$0.004 = 4 \times 10^{-3}$
Moved the decimal point 3 places to the left.	Moved the decimal point 3 places to the right.

In both cases, the decimal was moved 3 places to get the first factor, 4, by itself.

- The power of 10 is positive when the number is larger than $1\colon 4000 = 4 \times 10^{3}$.
- The power of 10 is negative when the number is between 0 and $1\colon 0.004 = 4 \times 10^{-3}$.

Example:

Exercise:

Problem: Write $37{,}000$ in scientific notation.

Solution:

Step 1: Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.	37000.
Step 2: Count the number of decimal places, n, that the decimal point was moved.	3.70000 4 places
Step 3: Write the number as a product with a power of 10.	3.7×10^4
If the original number is: • greater than 1, the power of 10 will be 10^n. • between 0 and 1, the power of 10 will be $10^{-\mathrm{n}}$	
Step 4: Check.	
10^4 is 10,000 and 10,000 times 3.7 will be 37,000.	
	$37{,}000 = 3.7 \times 10^4$

Example:
Exercise:

Problem: Write in scientific notation: 96,000.

Solution:

9.6×10^4

Note:
Try It
Exercise:

Problem: Write in scientific notation: 48,300.

Solution:

4.83×10^4

Note:
Convert from decimal notation to scientific notation

Move the decimal point so that the first factor is greater than or equal to 1 but less than $10.$

Count the number of decimal places, $n,$ that the decimal point was moved.

Write the number as a product with a power of $10.$

- If the original number is:
- greater than 1, the power of 10 will be 10^n.
- between 0 and 1, the power of 10 will be 10^{-n}.

Check.

Example:
Exercise:

Problem: Write in scientific notation: 0.0052.

Solution:

	0.0052
Move the decimal point to get 5.2, a number between 1 and 10.	0.0052
Count the number of decimal places the point was moved.	3 places
Write as a product with a power of 10.	5.2×10^{-3}

$$5.2 \times 10^{-3}$$
$$5.2 \times \frac{1}{10^3}$$
$$5.2 \times \frac{1}{1000}$$
$$5.2 \times 0.001$$
$$0.0052$$
$$0.0052 = 5.2 \times 10^{-3}$$

Example:
Exercise:

Problem: Write in scientific notation: 0.0078.

Solution:

7.8×10^{-3}

Note:
Try It
Exercise:

Problem: Write in scientific notation: 0.0129.

Solution:

1.29×10^{-2}

Convert Scientific Notation to Decimal Form

How can we convert from scientific notation to decimal form? Let's look at two numbers written in scientific notation and see.
Equation:

$$\begin{array}{ll} 9.12 \times 10^{4} & 9.12 \times 10^{-4} \\ 9.12 \times 10{,}000 & 9.12 \times 0.0001 \\ 91{,}200 & 0.000912 \end{array}$$

If we look at the location of the decimal point, we can see an easy method to convert a number from scientific notation to decimal form.

$$9.12 \times 10^{4} = 91{,}200 \qquad 9.12 \times 10^{-4} = 0.000912$$

$$9.12\_\,\_ \times 10^{4} = 91{,}200 \qquad \_\,\_\,\_9.12 \times 10^{-4} = 0.000912$$

In both cases the decimal point moved 4 places. When the exponent was positive, the decimal moved to the right. When the exponent was negative, the decimal point moved to the left.

Example:
Exercise:

Problem: Convert to decimal form: 6.2×10^{3}.

Solution:

Step 1: Determine the exponent, n, on the factor 10.	6×10^{3}
Step 2: Move the decimal point n places, adding zeros if needed.	6.200.
• If the exponent is positive, move the decimal point n places to the right. • If the exponent is negative, move the decimal point $\lvert n \rvert$ places to the left.	6,200

Step 3: Check to see if your answer makes sense.	
10^3 is 1000 and 1000 times 6.2 will be 6,200. $6.2 \times 10^3 = 6{,}200$	

Example:
Exercise:

Problem: Convert to decimal form: 1.3×10^3.

Solution:

1,300

Note:
Try It
Exercise:

Problem: Convert to decimal form: 9.25×10^4.

Solution:

92,500

Note:
Convert scientific notation to decimal form

Determine the exponent, n, on the factor 10.

Move the decimal n places, adding zeros if needed.	◦ If the exponent is positive, move the decimal point n places to the right. ◦ If the exponent is negative, move the decimal point $\lvert n \rvert$ places to the left.
Check.	

Example:
Exercise:

Problem: Convert to decimal form: 8.9×10^{-2}.

Solution:

	8.9×10^{-2}
Determine the exponent n, on the factor 10.	The exponent is -2.
Move the decimal point 2 places to the left.	$\_8.9$
Add zeros as needed for placeholders.	0.089
	$8.9 \times 10^{-2} = 0.089$
The Check is left to you.	

Example:
Exercise:

Problem: Convert to decimal form: 1.2×10^{-4}.

Solution:

0.00012

Note:
Try It
Exercise:

Problem: Convert to decimal form: 7.5×10^{-2}.

Solution:

0.075

Scientific Notation on a Scientific Calculator

It's important that you know how to enter numbers into a calculator that are given in scientific notation, without having to convert it to decimal notation first, as well as how to read an answer that the calculator gives in scientific notation.

On a scientific calculator, there is a button specifically meant for scientific notation. This button is either an Exp or an EE button. When you press the

button an E will show on the screen (or $\times 10^\wedge$) and the E represents $\times 10^\wedge$, where the ^ symbol means "raised to the". You would then follow that with the exponent.

Try with your calculator: Enter in 7.15×10^3 as 7.15 Exp (or EE) 3, hit enter or the = sign, and your calculator will show 7150, which is the decimal notation.

Now enter in 7.15×10^{15} as 7.15 Exp (or EE) 15, hit enter or the = sign, and your calculator should show 7.15×10^{15} or 7.15E15. This is because this number has too many digits to show on the screen in decimal notation.

Example:
Exercise:

Problem: Multiply $(1.5 \times 10^{-7})(8.2 \times 10^3)$ using your calculator.

Solution:

This would be entered into the calculator as (1.5 Exp (or EE) −7)(8.2 Exp (or EE) 3). The calculator should give 0.00123 as the answer.

Note:
Try It
Exercise:

Problem: Multiply $(9.9 \times 10^5)(9.9 \times 10^{20})$ using your calculator.

Solution:

Your calculator may show you 9.801 E 26, which means 9.801×10^{26}.

Example:
Exercise:

Problem: Use the calculator to divide: $\frac{2.04 \times 10^5}{3.4 \times 10^{-6}}$

Solution:

This is entered as 2.04 Exp 5 ÷ 3.4 Exp -6. The answer is 6×10^{10}.

Summary

- **Convert from Decimal Notation to Scientific Notation:** To convert a decimal to scientific notation:

  Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.
  Count the number of decimal places, n, that the decimal point was moved.
  Write the number as a product with a power of 10.
    - If the original number is greater than 1, the power of 10 will be 10^n.
    - If the original number is between 0 and 1, the power of 10 will be 10^n.

  Check.

- **Convert Scientific Notation to Decimal Form:** To convert scientific notation to decimal form:

  Determine the exponent, n, on the factor 10.
  Move the decimal n places, adding zeros if needed.
    - If the exponent is positive, move the decimal point n places to the right.
    - If the exponent is negative, move the decimal point $|n|$ places to the left.

Check.

Homework

Convert from Decimal Notation to Scientific Notation

In the following exercises, write each number in scientific notation.
Exercise:

Problem: 280,000

Solution:

2.8×10^5

Exercise:

Problem: 45,000

Exercise:

Problem: 1,290,000

Solution:

1.29×10^6

Exercise:

Problem: 8,750,000

Exercise:

Problem: 0.041

Solution:

4.1×10^{-2}

Exercise:

Problem: 0.036

Exercise:

Problem: 0.0000103

Solution:

1.03×10^{-5}

Exercise:

Problem: 0.00000924

Exercise:

Problem:

The population of the world on July 4, 2010 was more than 6,850,000,000.

Solution:

6.85×10^{9}

Exercise:

Problem:

The population of the United States on July 4, 2010 was almost 310,000,000.

Exercise:

Problem:

The probability of winning the 2010 Megamillions lottery is about 0.0000000057.

Solution:

5.7×10^{-9}

Exercise:

Problem: The average width of a human hair is 0.0018 centimeters.

Convert Scientific Notation to Decimal Form

In the following exercises, convert each number to decimal form.

Exercise:

Problem: 8.3×10^{2}

Solution:

830

Exercise:

Problem: 4.1×10^{2}

Exercise:

Problem: 1.6×10^{10}

Solution:

16,000,000,000

Exercise:

Problem: 5.5×10^{8}

Exercise:

Problem: 2.8×10^{-2}

Solution:

0.028

Exercise:

Problem: 3.5×10^{-2}

Exercise:

Problem: 6.15×10^{-8}

Solution:

0.0000000615

Exercise:

Problem: 1.93×10^{-5}

Exercise:

Problem:

At the start of 2012, the US federal budget had a deficit of more than $\$1.5 \times 10^{13}$.

Solution:

$15,000,000,000,000

Exercise:

Problem:

In 2010, the number of Facebook users each day who changed their status to ‘engaged’ was 2×10^{4}.

Exercise:

Problem: The width of a proton is 1×10^{-5} of the width of an atom.

Solution:

0.00001

Exercise:

Problem:

The concentration of carbon dioxide in the atmosphere is 3.9×10^{-4}.

Multiply and Divide Using Scientific Notation and a Scientific Calculator

In the following exercises, multiply or divide, using a scientific calculator, and write your answer in decimal form.

Exercise:

Problem: $(3 \times 10^{2})(1 \times 10^{-5})$

Solution:

0.003

Exercise:

Problem: $(2 \times 10^{5})(2 \times 10^{-9})$

Exercise:

Problem: $(2.1 \times 10^{-4})(3.5 \times 10^{-2})$

Solution:

0.00000735

Exercise:

Problem: $(1.6 \times 10^{-2})(5.2 \times 10^{-6})$

Exercise:

Problem: $\frac{8 \times 10^{6}}{4 \times 10^{-1}}$

Solution:

20,000,000

Exercise:

Problem: $\frac{6 \times 10^{4}}{3 \times 10^{-2}}$

Exercise:

Problem: $\frac{5 \times 10^{-3}}{1 \times 10^{-10}}$

Solution:

50,000,000

Exercise:

Problem: $\frac{7 \times 10^{-2}}{1 \times 10^{-8}}$

Rounding with Precision and Significant Figures

A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The “known masses” are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (credit: Serge Melki)

Learning Objectives

By the end of this section, you will be able to:

- Round answers from adding or subtracting measurements based on the least precise place value.
- Determine the appropriate number of significant figures to use in multiplication and division calculations.

Accuracy and Precision of a Measurement

Science is based on observation and experiment—that is, on measurements. The fields of health and medicine are also based heavily on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let's say that you are measuring the length of standard computer paper. The packaging in which you purchased the paper states that it is 11.0 inches long. You measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in. These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate.

The **precision** of a measurement system has to do with place value. For example, each of the measurements of the piece of paper are precise to the tenths place.

Here are some examples of measurements and their precision as shown by place values:

- 13.00 is precise to the hundredths place.
- 0.140 is precise to the thousandths place.
- 3400 is precise to the hundreds place.
- 239,000 is precise to the thousands place.
- 14 is precise to the ones place.

If there are no digits past the decimal, then the precision of the measurement is to the last place value with a nonzero digit, as you read the number from left to right. If there are digits to the right of the decimal, then the precision of the measurement is to the place value of the very last digit, as you read the number from left to right, even if the last digit is a zero.

Can you see how adding zeros past the decimal without reason can make a measurement more precise and dropping zeros will make a measurement less precise? If a measurement of 13.00, which is precise to the hundredths place, was written down as 13 or 13.0 or 13.000 then this would be wrong since it needs to have its last zero in the hundredths place exactly.

Note: Measurements are always rounded estimates because of the inaccuracy of the tool used. The idea of being an estimate must be reflected in the way the results are recorded.

The result of mathematical operations, performed with approximate numbers from measurements, is also an approximate number and must be rounded according to either the accuracy or the precision of the original numbers.

The output cannot be more accurate or precise than the input. This is the main idea! Again, **the output cannot be more accurate or precise than the input.**

Precision when Adding and Subtracting Measurements

When adding and subtracting measurements, the result must be rounded to the **least precise place value** of the measurements involved. The key is that **the result cannot be any more precise than the least precise measurement given.**

When given two or more measurements to add (or subtract), first add as normal. In other words, **do not round the numbers before you add**. Then, after adding (or subtracting), round the result to the least precise place value of the given numbers.

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

Example:
Exercise:

Problem: Add: 15,800 mi. + 11,000 mi.

Solution:

We first add the numbers to get 26,800 mi.

Next we note that 15,800 is precise to the hundreds place and 11,000 is precise to the thousands place.

The thousands place is less precise than the hundreds place, so we round the result to the thousands place.

Thus, the final answer is 27,000 mi. (Notice how the units of miles is included.)

Note:
Steps for adding and subtracting measurements with precision

Make sure the measurements are in the same units. (We can’t add km to m, for example)
Add or subtract the numbers as they are given to you.
Round the result to the least precise place value.
Label the result with the correct units. Don't forget this very important step!

Example:
Exercise:

Problem: Add: 58.03 m + 11 m

Solution:

First add the numbers to get 69.03 m.

Then, note that 58.03 is precise to the hundredths place and 11 is precise to the ones place.

The hundredths place is more precise then the ones place, so we need to round the result to the ones place.

Thus, the final answer is 69 m.

Note:
Try It
Exercise:

Problem:

Add 24.3 ft. and 16 ft. Round the result to the least precise place value.

Solution:

The answer is 40 ft. Note that this result is precise to the tens place due to the zero in the ones place. However, the key is that the result cannot be any more precise than the least precise place value of the measurements given, which indeed is the case here.

Example:
Exercise:

Problem: Subtract 344 cm from 48 m.

Solution:

First, we must convert these into the same units. We can either convert the centimeters to meters or convert the meters to centimeters.

If we convert the meters to centimeters, then this will give us a subtraction problem of 4,800 cm - 344 cm.

Next, we do the subtraction, which gives us 4,456 cm.

Note that 4,800 is precise to the hundreds place and 344 is precise to the ones place. Since the hundreds place is less precise than the ones place, we round our answer to the hundreds place.

The final answer is 4,500 cm.

Precision when Multiplying and Dividing Measurements, Using Significant Figures

Because measurements are approximations they are rounded estimates. When multiplying or dividing by a rounded number, the result must be rounded correctly to reflect the estimation. This is done by considering the significant digits of each number.

For multiplication and division: The result should have the same number of significant figures as the measurement having the least significant figures entering into the calculation.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool. For example, if you use a standard ruler to measure the length of a stick, you may measure it to be 36.7 cm. You could not express this value as 36.71 cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36.6 cm and

$36.7\ \mathrm{cm}$, and he or she must estimate the value of the last digit. Using the method of **significant figures**, the rule is that *the last digit written down in a measurement is the first digit with some uncertainty*. In order to determine the number of significant digits in a value, start with the first measured value at the left and count the number of digits through the last digit written on the right. For example, the measured value $36.7\ \mathrm{cm}$ has three digits, or significant figures. Significant figures indicate the precision of a measuring tool that was used to measure a value.

Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant, because they are only placeholders that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placeholders, but are significant — this number has five significant figures. The zeros in 1300 may or may not be significant depending on the style of writing numbers. They could mean the number is known to the last digit, or the zeros could be placeholders. So 1300 could have two, three, or four significant figures. (To avoid this ambiguity, write 1300 in scientific notation.) ***Zeros are significant except when they serve only as placeholders***.

Rules for identifying and counting significant digits.
Digits that are **significant** are:

- All nonzero digits, such as in 563, which has 3 significant digits.
- All embedded zeros, such as in 7,500.06, which has 6 significant digits.
- Trailing zeros after the decimal point, such as in 2.4500, which has 5 significant digits.

Digits that are **not significant** are zeros merely used as place holders between other digits and the decimal point or isolated zeros in front of the decimal point:

- Leading zeros, such as in 0.000008, which has only 1 significant figure.
- Trailing zeros before the decimal point, such as in 16000, which has 2 significant digits.

Exercise:
Check Your Understanding

Problem:

Determine the number of significant figures in the following measurements:

a. 0.0009
b. 15,450.0
c. 87.990
d. 30.42
e. 2,500,000
f. 6.20×10^3

Solution:

(a) 1; the zeros in this number are placeholders that indicate the decimal point

(b) 6; here, the zeros indicate that a measurement was made to the 0.1 decimal point, so the zeros are significant

(c) 5; the final zero indicates that a measurement was made to the 0.001 decimal point, so it is significant

(d) 4; any zeros located in between significant figures in a number are also significant

(e) 2; the zeros here are merely placeholders

(f) 3; the zero here shows precision so should be counted, where as 6.2 × 10^3 would only have 2 significant figures

Steps for rounding results of multiplication or division, using significant digits:

It is okay to have different units or labels, except for some circumstances, like calculating volume.
Multiply or divide the given numbers.
Round the result to the lowest count of significant digits from the given numbers.
Label the result correctly with proper units.

Example:
Exercise:

Problem:

The area of a circle can be calculated from its radius using $A = \pi r^2$.

Let's see how many significant figures the area has if the radius has only two—say, $r = 1.2$ m. Then,

Solution:
Equation:

$$A = \pi r^2 = (3.1415927...) \times (1.2\ \text{m})^2 = 4.5238934\ \text{m}^2$$

is what you would get using a calculator that has an eight-digit output. But because the radius has only two significant figures, it limits the calculated quantity to two significant figures or
Equation:

$$A{=}4.5\ \text{m}^2,$$

even though π is good to at least eight digits.

Example:
Exercise:

Problem:

Multiply 458.3 cm × 0.002 cm. Round the answer using significant digits.

Solution:

Using a calculator, we get 458.3 × 0.002 = 0.9166. Since 0.002 only has one significant digit (all the zeros are placeholders), we must round the answer to 0.9. The final answer, including units, is 0.9 cm^2.

Example:
Exercise:

Problem:

Divide: 850 mL ÷ 60 min. Round the result using significant digits.

Solution:

Using the calculator, 850 ÷ 60 = 14.66666... Since 60 has only one significant digit, which is the lowest number of significant digits between the two numbers, so must the answer. Thus, the final answer, including units is 10 mL/min.

Note:
Try It

Exercise:

Problem:

Divide: 45.6 yd^2 ÷ 9.1 yd. Round the result using significant figures.

Solution:

Using a calculator, 45.6 ÷ 9.1 = 5.010989011... Since 9.1 has the smallest number of significant digits, namely 2, so must the answer. Thus, the final answer, including units, is 5.0 yd.

Significant Figures in this Text

In this text, you only need to use these rules when specifically asked to do so. Furthermore, other examples and solutions to homework exercises will not necessarily follow these same rules. Whether you need to follow the rules of rounding with precision in your career will depend on your field.

Summary

- Accuracy of a measured value refers to how close a measurement is to the correct value.
- Precision of measured values is based on the last place value.
- The precision of a *measuring tool* is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.
- Significant figures express the precision of a measuring tool.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the value with the least number of significant figures.

Homework

Express your answers to problems in this section to the correct number of significant figures or least precise place value and remember to include proper units.

Exercise:

Problem: Add 0.08 g and 132 mg.

Solution:

210 mg or 0.21 g

Exercise:

Problem: Find the difference: 4.05 L – 1.9 L.

Exercise:

Problem: Multiply: 6.1 m × 8 m × 5.02 m

Solution:

200 m^3

Exercise:

Problem: Multiply: 27 miles × 3.0 miles

Exercise:

Problem: Multiply: 12.6 kg × 0.25 hr.

Solution:

3.2 kg-hr.

Exercise:

Problem: Calculate: 56.4 m + 23 m – 10 m

Exercise:

Problem: Subtract: 1098 mi. – 940 mi.

Solution:

160 mi.

Exercise:

Problem: Subtract: 62.5 yd. − 8 yd.

Exercise:

Problem: 25 milligrams divided by 12 pills

Solution:

2.1 mg/pill

Exercise:

Problem: Multiply: (3.21 cm)(4.1 cm)

Logarithms

Learning Objectives

By the end of this lesson, you will be able to:

- Understand the meaning of a logarithm (log for short).
- Determine the log of a power of ten, without a calculator.
- Use a scientific calculator to find the log of other numbers.
- Use a scientific calculator to find the log of a number in scientific notation.

Logarithms are exponents. When we write a number in the power of ten form, we can see the **log** as it is the exponent on the ten. (For this course, we will only consider logs on the base of ten.)

For example, log(1000) = 3, since 1000 = 10^3

Note: log(1000) is read "log of 1000".

Example:
Exercise:

Problem: Find the log of each power of 10, without a calculator.

a. 0.001
b. 100,000
c. 10

Solution:

a. log(0.001) = -3, since 0.001 = 10^{-3}
b. log(100,000) = 5, since 100,000 = 10^5

c. log(10) = 1, since 10 = 10^1

What is the log of 212? We can’t do this one without a calculator since it's not a power of 10 with an integer exponent. However, since 212 is between 100 and 1000, We can estimate that its log will be between the log of 100 and the log of 1000, which means it’s between 2 and 3.

We can use a key on the calculator, LOG, to find it. Enter in LOG 212 to see that is equals about 2.326. Notice how this is in fact between 2 and 3.

Why is the log of 212 equal to about 2.326? Because 10 to the exponent of 2.326 is about 212. Check this using the y^x or ^ key. See that $10^{2.326}$ = 211.836, which is close to 212. (It is off a little due to the rounding.)

Because all positive numbers can be written as a power of ten, we can find the log of all positive numbers. Logs are used in other studies to make calculations easier, but for this course we will use logs for pH applications.

Example:
Exercise:

Problem:

Use the log button on your scientific calculator to find the following logs:

a. log(550)
b. log(0.94)
c. log(-50)

Solution:

a. about 2.74
b. about -0.0269
c. This will give you an error. We can only find the log of positive numbers since you can't raise 10 to any exponent and get a negative number as a result.

Example:
Exercise:

Problem:

Find the log of the following numbers. Remember to use your EE or Exp button to enter numbers that are in scientific notation.

a. 3.25×10^{-4}
b. 2.1×10^{4}
c. 6.25×10^{-2}

Solution:

a. about -3.488
b. about 4.32
c. about -1.20

Homework

For the following exercises, determine the log **without a calculator**.
Exercise:

Problem: $\log(10^{-7})$

Solution:

-7

Exercise:

Problem: $\log(10^{13})$

Exercise:

Problem: log(1,000,000,000)

Solution:

9

Exercise:

Problem: log(100,000)

Exercise:

Problem: log(0.0000001)

Solution:

-7

Exercise:

Problem: log(0.00001)

For the following exercises, use a scientific calculator to find the log. Round each answer to 3 decimal places.

Exercise:

Problem: log(935)

Solution:

2.971

Exercise:

Problem: log(2762)

Exercise:

Problem: log(0.00325)

Solution:

-2.488

Exercise:

Problem: log(0.00467)

Exercise:

Problem: $\log(9.06 \times 10^{17})$

Solution:

17.957

Exercise:

Problem: $\log(1.18 \times 10^{13})$

Exercise:

Problem: $\log(8.002 \times 10^{-15})$

Solution:

-14.097

Exercise:

Problem: $\log(3.204 \times 10^{-23})$

pH and Molarity

Learning Objectives

By the end of this lesson, you will be able to:

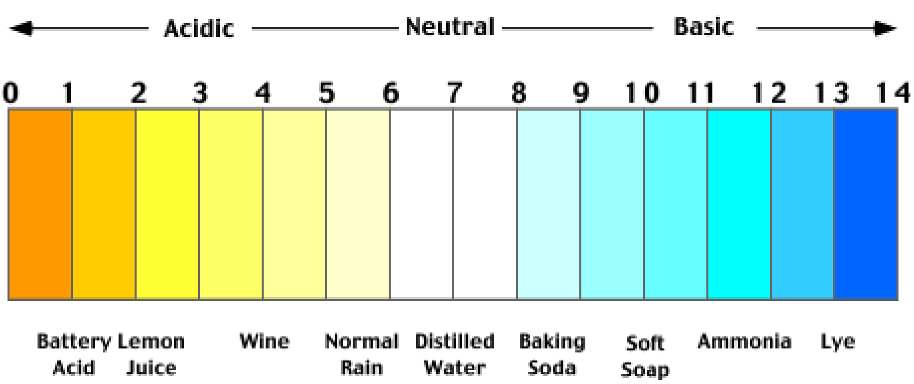
- Understand the meaning of pH.
- Understand the formula for finding pH, based on molarity.
- Calculate pH of a solution given its molarity.

A solution may contain acids which produce hydrogen ions or a base which decreases the hydrogen ions. The acidity or base of the solution can be determined by the amount of hydrogen ions present. The measurement of the hydrogen ion concentration is equal to the molarity of the solution.

Molarity is a rate of " moles per liter." Molarity (M) is used to measure the concentration of hydrogen ions in a solution, which is used with pH applications.

The molarity is usually written in scientific notation, for example 5.5×10^{-4} M means there is a partial mole (0.00055) of particles in one liter for this particular solution.

It is common to use **pH** to tell if a solution is acidic or basic. The pH scale shows a reading of 7 as neutral, a reading from 0 to below 7 as acidic and a reading from above 7 to 14 as basic.



The pH formula
pH formula: pH = −log[H+] (This formula will be given on tests.)

Because the scientific notation form for molarity is difficult to use directly as a basis for determining acidity, this formula changes it into a standard number between 0 and 14, which is the pH level. Let's analyze the formula reading it from right to left this time before we enter it into our calculator.

[H+]

The [H+] is the molarity number, which shows the concentration of hydrogen ions in the solution. This will be a small decimal number written in scientific notation so it will have a negative exponent.

LOG

The log part of the formula will give us a negative number because the molarity reading is always a decimal number (between 0 and 1). Remember that logs are the exponents on the base of ten so a molarity reading of 2.65×10^{-5} M will give us a log that is a little more than −5. Enter in LOG 2.65 EXP −5 = to get about −4.576754.

Opposite sign "−" (**Caution** this is NOT the subtraction key)

This part of the formula makes the final number positive. pH is usually rounded to the hundredths place. So -(−4.576754) becomes 4.58. Thus, the pH of a solution with a molarity of 2.65×10^{-5} M is 4.58. We can see from the scale that this solution is slightly acidic.

Entering the formula into the calculator:

This formula uses "–" for opposite or negative so don't use the subtraction sign.

It uses EXP or EE for the scientific notation form.

It uses the LOG key.

Example:
Exercise:

Problem:

Given a molarity of 4.32×10^{-5} M for hydrogen ions in a solution, find the pH with the given formula.

Solution:

Enter in: − LOG 4.32 Exp −5 = to see about 4.3645 on the screen. Round to 4.36 for pH.

Example:
Exercise:

Problem:

Find the pH of the solution that has hydrogen ions of 3.67×10^{-8} M.

Solution:

Enter in: –LOG 3.67 EE −8 = 7.43533. Round to 7.44 for pH.

Homework

For the following exercises. Use the pH formula and a scientific calculator to determine the pH level.
Exercise:

Problem:

Find the pH of the solution that has a hydrogen ion concentration of 1.26×10^{-5} M.

Solution:

pH = 4.90

Exercise:

Problem:

Find the pH of a solution that has a molarity of 4.6×10^{-3} M.

Exercise:

Problem:

Find the pH of a solution that has a molarity of 6.84×10^{-12} M.

Solution:

pH = 11.16

Exercise:

Problem:

Find the pH of sauerkraut with a hydrogen ion concentration of 3.162×10^{-4} M.

Exercise:

Problem:

Find the pH of milk with a hydrogen ion concentration of 2.512×10^{-7} M.

Solution:

pH = 6.60

Exercise:

Problem:

Find the pH of baking soda with a hydrogen ion concentration of 3.1×10^{-9} M.